the enclosed region was at ambient pressure and the observed lift coefficient was 0.41.

The model originally used to eliminate the particle forces (Fig. 1) was a single sphere resting on a bed of equal-size spheres. In view of the forementioned data, it is apparent that the pressure on the surface in contact with the bed will be near the ambient-stream static pressure. Since the gross interference effects of the wall are negligible, the particle forces can be estimated using the solution for a sphere in a nonuniform flow. The method used is to assume that solution for the pressure distribution on a sphere in uniform shear flow[5] applies up to the point where the sphere contacts the bed, and to assume that the pressure on the surface in contact with the bed is the ambient static pressure. For $k = (a/u_0) \times (\partial u/\partial y)$, the tangential velocity on the sphere is, to the second order in k,

$$v/u_0 = \tfrac{3}{2} \sin\theta + k(2 \sin^2\theta - 1.3505) + 0.967k^2 \sin\theta + \ldots \quad (1)$$

The lift on the sphere follows directly from the foregoing assumptions:

$$C_L = 6k \left\{ \left(\frac{8}{5} - A\right)\left[1 - \cos\theta_1 - \frac{1}{3}(1 - \cos^3\theta_1)\right] - \frac{2}{5}\sin^4\theta_1 \cos\theta_1 \right\} + \frac{2}{\pi}\int_{\theta_1}^{\pi/2} \Big\{ 2(B \sin^2\theta - 1) \times (\sin^2\theta - \sin^2\theta_1)^{1/2} \sin\theta + 3k(2\sin^2\theta - A) \times \left[\frac{\pi}{2} + \sin^{-1}\left(\frac{\sin\theta_1}{\sin\theta}\right) - \frac{\sin\theta_1}{\sin^2\theta}(\sin^2\theta - \sin^2\theta_1)^{1/2}\right]\sin^3\theta + 2k^2 \sin\theta\,(\sin^2\theta - \sin^2\theta_1)^{1/2}\left[\frac{2}{3} + \frac{1}{3}\frac{\sin^2\theta_1}{\sin^2\theta}\right] \times [A^2 - (4A - C)\sin^2\theta + 4\sin^4\theta] \Big\} d\theta \quad (2)$$

where $A = 1.3505$, $B = 2.25$, $C = 2.901$, and θ_1 is the absolute magnitude of θ where the sphere contacts the bed.

There are two important items to be noted in Eq. (2). The limit of $\theta_1 = \pi/2$ corresponds to the sphere in uniform shear flow and yields a lift coefficient, $C_L = 0.998k$. This value differs from that given in Ref. 1 since the latter was obtained with a stripwise integration. Second, it should be noted that there is no second-order effect of shear on the lift owing to the symmetry in Eq. (1).

Equation (2) has been integrated numerically for the model illustrated in Fig. 1, i.e., for $\theta_1 = 55°$. The resulting lift coefficient is

$$C_L = 0.345 + 0.557k + 0.879k^2 \quad (3)$$

This result has been applied following the method described in Ref. 1 to arrive at approximate criteria for particle entrainment. The one important difference in this computation is to note that one must define the position of an equivalent smooth wall. The wall position arbitrarily is assumed to be where the sphere contacts the bed. The resulting criteria for particle entrainment are given in Fig. 1. The interpretation of these results is identical to that for Fig. 7 in Ref. 1. The important item to be noted is that lift entrainment is predicted for a wider range of particle sizes and flow conditions than in Ref. 1. Similarly, drag entrainment is predicted for a smaller range of conditions, owing to the assumption as to the effective position of the boundary layer base. Included in Fig. 1 are the particle sizes for which the shear k exceeds unity. For $k \geq 1$, the theory of Ref. 5 ceases to be valid.

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Errata and Addendum: "A Second-Order Theory of Entry Mechanics into a Planetary Atmosphere"

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REFERENCE 1 was published when the author was in Bulgaria to attend the XIIIth International Congress of Astronautics. Because of lack of author's proofreading, several printing errors were found in the paper. Therefore, the following errata should be made immediately.

I. Mismatch between Fig. Nos. and Fig. Captions

The figures were renumbered by the editorial staff, whereas the captions corresponding to each originally numbered figure were retained by the printer. This results in 1) mismatch between the figure and the caption in each figure, and 2) mismatch between the figure number with the figure number referred to in the main text.

In order to match the right figure with the right caption and to match the right figure number with the figure number referred to in the main text, the following errata should be made:

1) Figures 7c and 7d: "Comparison of gliding entry at large initial angle of inclination ($\theta_f = 12°$) and for $L/D = 1.0$" should be corrected to read as Figs. 3a and 3b: "Comparison of complete spectrum of L/D gliding entry at small initial angles of inclination ($\theta_f \cong 0$)."

2) Figures 3a and 3b: "Comparison of complete spectrum of L/D gliding entry at small initial angles of inclination ($\theta_f \cong 0$)" should be corrected to read as Figs. 4a and 4b: "Comparison of ballistic entry at small and large initial angles of inclination θ_f."

3) Figures 4a and 4b: "Comparison of ballistic entry at small and large initial angles of inclination θ_f" should be corrected to read as Figs. 5a and 5b: "Comparison of the effect of $W/(C_DA)$ ten times larger and ten times smaller."

4) Figures 5a and 5b: Comparison of the effect of $W/(C_DA)$ ten times larger and ten times smaller" should be corrected to read as Figs. 6a and 6b: "Comparison of the effect of βR_0 ten times larger and ten times smaller."

5) Figures 6a and 6b: "Comparison of the effect of βR_0 ten times larger and ten times smaller" should be corrected to read as Figs. 7a and 7b: "Comparison of gliding entry at large initial angle of inclination ($\theta_f = 12°$)."

6) Figures 7a and 7b: "Comparison of gliding entry at large initial angle of inclination ($\theta_f = 12°$)" should be corrected to read as Figs. 8a and 8b: "Comparison of supercircular-velocity entry at large initial angle of inclination ($\theta_f = 12°$)."

7) Figures 8a and 8b: "Comparison of supercircular-velocity entry at large initial angle of inclination" should be corrected to read as Figs. 7c and 7d: "Comparison of gliding entry at large initial angle of inclination ($\theta_f = 12°$)."

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8) The table on p. 1217 should read as follows:

First-order theories	Fig. no.
(1) Allen and Eggers	Fig. 3
(2) Chapman	Fig. 3
(3) Gazley	Fig. 4
(4) Lees (Ting)	Figs. 7c, 7d
(5) Loh	Figs. 3, 7
(6) Previous solution unavailable	Figs. 3, 4
(7) Supercircular	Figs. 8a, 8b, 8c, 8d

II. Misprinting

1) The symbol ζ contained in Eqs. (14b, 21, and 23–25) should be the symbol ρ.

2) The numerator of the right-hand term of the first equation appearing on the right-hand side of p. 1215 should read $\theta_f - \theta$ instead of $\theta - \theta_f$.

3) The numerator of the right-hand term of the second equation appearing on the right-hand side of p. 1215 should read $(L/D)\ \ln\ \{1/[V^2/(gR_0)]\}$ instead of $(L/D)\ \ln\ \{1/[V_2/(gR_0)]\}$.

4) The Fig. 8 mentioned in the text on the right-hand side of p. 1216 should be Fig. 9, which is the figure contained in Ref. 2.

III. Addendum

Since the submission of Ref. 1, Refs. 2 and 3 were written. References 2 and 3 show that the second-order solution also may be reduced analytically to the previous solutions given by Arthur and Karrenberg and Wang and Ting in Refs. 4 and 5. These references are in addition to the numerical checks presented in Ref. 1.

References

[1] Loh, W. H. T., "A second-order theory of entry mechanics into a planetary atmosphere," J. Aerospace Sci. **29,** 1210–1222 (1962).

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Comments on "Calculation of Laminar Separation"

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In a recent paper, Morduchow and Reyle[1] reported a formula, due to Morduchow and Clarke,[2] by means of which the position of separation of a laminar boundary layer may be calculated in incompressible flow or in compressible flow with zero heat transfer. Calculated results were presented for two families of solutions as follows:

1) Compressible flow with zero heat transfer, $\sigma = 1$, $\mu \alpha T$, and $u_1 = u_\infty(1 - \xi)$, the values of ξ at separation being calculated at various Mach numbers.

2) Incompressible flow, with $u_1 = u_\infty(1 - \xi^n)$, the values of ξ at separation being calculated for various values of n.

The purpose of the present note is to indicate that the conclusions reached by Morduchow and Reyle are somewhat misleading. Begin with the case of compressible flow with zero heat transfer, where comparisons are made with the calculations of Stewartson,[3] showing remarkable agreement over a range of Mach numbers M_∞ from 0 to 10. The implication that the proposed formula is thus exceedingly accurate is, however, fallacious. It does not appear to be very widely known that these early results of Stewartson show an error that increases with Mach number. In particular, when $M_\infty = 4$, the results tabulated by Morduchow and Reyle indicate that $\xi_s \underline{\Omega}\ 0.06(0)$, according to either their own method or Stewartson's. However, numerical solutions of the boundary layer equations for this problem by Mathematics Division, National Physical Laboratory, reported by Curle[4] show that $\xi_s \underline{\Omega}\ 0.04(5)$, with an error of under 10%. The values of ξ_s given by Stewartson and by Morduchow and Reyle accordingly are alike in error by about 30 or 40%.

With regard to the case of incompressible flow, it may be remarked that a method exists which is both simpler to apply and more accurate in its predictions than the method under discussion. It was originally due to Stratford[5] and was simplified somewhat by Curle and Skan.[6] In its simplest form, the method gives the values of ξ at separation by solution of the algebraic equation

$$C_p[\xi(dC_p/d\xi)]^2 = 0.0104$$

where $C_p = 1 - (u_1^2/u_\infty^2)$ is the pressure coefficient.

The simplicity is apparent, and the accuracy is indicated in Table 1.

It will be noted that the error is only 10% of that given by Morduchow and Reyle. The method of Stratford has been applied to the various other cases for which essentially exact solutions of the laminar boundary layer equations are available (Terrill[7] and Curle[8]) and the considerable accuracy verified.

References

[1] Morduchow, M. and Reyle, S. P., "On calculation of the laminar separation point, and results for certain flows," J. Aerospace Sci. **29,** 996 (1962).

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Table 1 Separation point for $u_1/u_\infty = 1 - \xi^n$, $M_\infty = 0$

n	1	2	4	8
ξ_s (Howarth and Tani)	0.120	0.271	0.462	0.641
ξ_s (Morduchow and Reyle)	0.122	0.268	0.452	0.625
ξ_s (Stratford)	0.121	0.271	0.461	0.639

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