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BIAXIAL PROPERTIES OF METALS FOR AEROSPACE APPLICATIONS

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BIAXIAL PROPERTIES OF METALS FOR AEROSPACE APPLICATIONS

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Abstract

The following properties of thin metallic members under biaxial loading conditions are discussed: stress-strain curves, yield strength, and ultimate strength. Emphasis is placed on information useful in the design of primary structures of aerospace vehicles, liquid-propellant tankage, and solid-propellant motor cases. Three criteria for biaxial yield strength, comparable to the 0.2 per cent offset criterion for uniaxial yield strength, are discussed. Then a new generalized equation for describing the biaxial yield-strength envelope is presented. As an example, biaxial data for AISI 4340 steel heat treated to various strength levels are used.

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Introduction

As more advanced aerospace vehicles are developed, biaxial loading is becoming increasingly important. This is invariably true in the case of pressure vessels, including high-pressure fluid-storage bottles, pressurized cabins, solid-propellant rocket-motor cases, and liquid-propellant tankage. However, it is also important in certain aerodynamic surfaces as well, such as low-aspect-ratio wings.

Under static conditions, a structure may lose its strength; i.e., failure may occur, by one of the following mechanisms:

- Brittle fracture
- Buckling
- Yielding
- Plastic instability, the phenomenon which determines ultimate strength.

In the past 5 years so much difficulty has been experienced with brittle fracture of so-called high-strength materials, particularly in solid-propellant rocket-motor cases, that considerable attention has been directed to this problem. As a result of more sound design, improved materials, better methods of fabrication and testing, and more careful inspection procedures, the brittle-fracture problem has been substantially reduced in severity. This has permitted ordinary yield-strength and ultimate-strength values to be reached in many recent instances.

In the past, it has been adequate to use uniaxial-strength properties for the design of biaxially loaded structures. However, as weight requirements have become more stringent, the increase in strength which many ductile metals exhibit under certain biaxial conditions as compared to uniaxial strength values has been looked at with increasing interest. Although according to the well-known octahedral-shear-stress theory[2] this increase in strength amounts to as much as approximately 15-1/2 per cent, some materials do not exhibit any increase. Furthermore, certain materials exhibit an appreciable decrease in strength under certain biaxial-loading conditions.

In view of the factors described above, there is considerable current interest in detailed experimental data on biaxial strength properties. Although the methods described in this paper are equally as applicable to strength data in the cryogenic and elevated-temperature regimes as they are to room-temperature data, the latter are used in the illustrative example.

The plastic instability phenomenon, which determines maximum load-carrying capacity or ultimate strength under biaxial as well as uniaxial loading is not discussed here since it has been treated for various geometrical configurations by Sachs and Lubahn (1)[3]. They showed theoretically that there is an appreciable geometrical effect. For example, they demonstrated that there can be a considerable difference between the ultimate strength of a thin-walled pressurized sphere and that of a thin, flat sheet loaded uniformly in the plane of the sheet. Also recent theoretical work such as that of Mann-Nachbar and Hoffman (2) as well as experimental data (3) indicates that plastic instability in thin-walled cylindrical pressure vessels is highly dependent upon the length-to-diameter ratio. However, research is being directed toward development of reliable design data on ultimate strength of simple thin-walled shells made of various aerospace metals.

In order to determine standardized yield-strength values for various biaxial-loading conditions, it is first necessary to develop an appropriate criterion of yielding similar to the widely recognized 0.2 per cent offset-strain criterion for uniaxial yield strength. Once this has been decided upon, it is advantageous to have a satisfactory data-reduction method to fit a smooth curve through the experimental data points, which are usually limited to a few biaxial-loading conditions.

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[2] This theory is described in detail later.

[3] Numbers in parentheses indicate references at end of paper.

The subject of this paper is the development of a biaxial yield criterion for metals and a new method for reducing biaxial-yield-stress data. In this paper the following aspects of biaxial properties of metals are discussed:

- Presentation of biaxial stress-strain data
- Biaxial yield criteria
- Presentation and reduction of biaxial yield-strength data.

Some Basic Definitions

The following definitions concerning multiaxial stress states are presented by way of review.

Normal stresses are direct stresses acting normal to a given cross-sectional area; they are shown acting on a typical element of a flat sheet or thin-walled structure in Figure 1-a as solid-line arrows. They are to be distinguished from shear stresses, shown as dotted-line arrows in Figure 1-a which are oriented in a direction parallel to the cross-sectional area on which they act.

If one properly selects the direction of the element which is cut from the structure, there is always some orientation[4] in which there is no shear stress acting on any face of the element. The directions of the faces of such an element are called principal directions and the stresses acting on these faces are known as principal stresses. In the case of two-dimensional or biaxial loading one principal stress, the one in the thickness direction, is identically zero and the two nonzero principal stresses lie in the plane of the sheet and always act perpendicular to each other as shown in Figure 1-b. The numerically larger of the two nonzero principal stresses is called the maximum principal stress.

If one selects three mutually perpendicular planes through a point, there is always some orientation of this system such that only normal strains exist, all shearing strains being zero. These normal strains are known as principal strains. The directions of the principal strains coincide with the directions of the principal stresses only for isotropic materials.

The state of stress in a biaxial-loading situation is completely defined by the biaxial ratio B, defined as the ratio of one principal stress to the other, the latter acting in a principal direction arbitrarily selected as the reference principal direction.

In the case of components in the form of a tube, it is conventional to use the circumferential direction as the reference; then the other biaxial stress is in the axial direction. Obviously, a tube which is extruded would have higher strength values in the

[4] The exact orientation is not of concern here. However, procedures for obtaining it, as well as the numerical values of the principal stresses, from strains measured by means of strain-gage rosettes are well known to stress analysts. Reference is made to books on elementary stress analysis or strength of materials.

axial direction and would give quite different results than those for a tube fabricated from a rolled sheet with the rolling direction oriented circumferentially, so care must be taken in specifying the method of fabrication, direction of rolling, etc. For a flat sheet, the terms circumferential and axial have no meaning, so in this case, the terms longitudinal and transverse should be used instead.

Biaxial Stress-Strain Relations

It is a fundamental assumption in current material-behavior studies that the stress-strain relations of a particular material in a given condition, i.e. prior thermal and mechanical history, are uniquely determined by the specimen size, the stress state, the temperature, and the rate of loading. This assumption has been substantiated experimentally provided there is only one loading beyond the elastic range, so that the Bauschinger effect does not take place (4). Thus, the stress-strain curves corresponding to various biaxial ratios in uniformly loaded thin-sheet or thin-walled specimens can be considered to be fundamental material properties just as is the uniaxial tensile stress-strain curve.

Typical biaxial stress-strain curves obtained by Goodman (5) are presented in Figure 2 in the format proposed for MIL-HDBK-5 (Military Handbook 5). The data are presented as maximum principal stress versus maximum principal strain and they cover only positive biaxial ratios (i.e., tension-tension loading, rather than tension-compression loading), which are of primary interest in most current applications on advanced aerospace vehicles.

In Figure 2 it is noted that within the elastic range the biaxial elastic modulus is slightly different for each biaxial ratio. This is explained as follows: the biaxial stress-strain relations for an isotropic elastic material (6) can be written as:

$$e_x = (f_x - \mu f_y)/E = (1 - \mu B)f_x/E,$$

$$e_y = (f_y - \mu f_x)/E = (B - \mu)f_x/E,$$

where E is the ordinary uniaxial modulus of elasticity, μ is the elastic value of Poisson's ratio, e_x and e_y are the principal strains, f_x and f_y are the principal stresses, and B is the biaxial ratio.

Solving these simultaneous equations for f_x and e_y, respectively, in terms of e_x, gives:

$$f_x = (e_x + \mu e_y)E/(1 - \mu^2),$$

$$e_y = e_x\,(B - \mu)/(1 - \mu B).$$

Substituting e_y from the latter equation into the former equation and simplifying the following result:

$$f_x = \left[E/(1 - \mu B)\right] e_x. \qquad [1]$$

Assuming the x direction to be the reference principal direction, the biaxial elastic modulus for an isotropic material at a ratio B is the quantity $E/(1 - \mu B)$. As an example, for a Poisson's ratio of 0.30 and a biaxial ratio of 1.0, the biaxial elastic modulus is approximately 43 per cent higher than the uniaxial modulus.

The plastic strain refers to the plastic portion of the total strain, i.e. the total strain minus the elastic strain. Use of the plastic deformation itself, rather than its incremental value generally implies use of the deformation theory of plasticity rather than the flow (or incremental) theory. The flow theory is generally conceded to be more correct; however, for the usual situation of continuous loading at a constant biaxial ratio, the two theories coincide. Furthermore, even for situations when these conditions are not met, for the small strains associated with yield criteria, the difference between the deformation and the incremental deformation is usually not very large.

Biaxial Yield Criteria

The purpose here is to discuss the reasoning behind the development of several criteria for isotropic or nearly isotropic metals under biaxial loading. All of these are comparable to the well-known 0.2 per cent offset yield-strength criterion for uniaxial loading. It is desirable that a standard biaxial yield criterion be selected so that there is unanimity in analyzing and presenting data as well as in design use.

The three yield criteria discussed here are:

- Uniform plastic-strain criterion
- Equivalent plastic-strain criterion
- Equivalent plastic-work criterion.

Perhaps the most simple biaxial yield criterion is the uniform plastic-strain criterion. The reasoning here is that if 0.2 per cent permanent strain is acceptable for uniaxial loading, it should also be acceptable under biaxial loading conditions. Although this criterion has been used by several aerospace companies, objections have been raised to its use because it is independent of the biaxial ratio.

In applying the uniform plastic-strain criterion to an appropriate biaxial stress-strain curve, such as shown in Figure 2, a parallel straight line is constructed from the point representing 0.2 per cent strain to its intersection with the appropriate stress-strain curve. The slope of the line is the same as that of the elastic portion of the biaxial stress-strain curve. The stress value at which the offset line intersects the stress-strain curve is the yield strength.

For a wide variety of structural metals, it has been found experimentally that under multiaxial loading the initiation of yielding occurs when a certain "effective stress" reaches a value equal to the uniaxial yield strength in tension. For most structural metals, this effective stress is the stress value associated with the von Mises yield criterion, which is the same as the octahedral-shear-stress theory, the distortion-energy theory[5], or improperly, the deformation-energy theory[6]. The general expression for the von Mises effective stress $\bar{f}$ is

$$\bar{f} = (1/\sqrt{2})\sqrt{(f_x - f_y)^2 + (f_y - f_z)^2 + (f_z - f_x)^2}, \qquad [2]$$

where f_x, f_y, and f_z are the three principal stresses. For the biaxial-loading case, which is the primary interest here, Equation [2] reduces to

$$\bar{f} = \sqrt{f_x^2 - f_x f_y + f_y^2}. \qquad [2a]$$

For practical engineering purposes, we are interested in the stress corresponding to some plastic (offset) strain, such as 0.2 per cent, rather than the stress at which yielding begins. Since the effective stress concept was well established, it was a logical step, first made by Dorn and Thomsen (7), to define an effective plastic strain $\bar{e}_p$ by an equation analogous to the von Mises criterion, namely[7],

$$\bar{e}_p = (\sqrt{2}/3)\sqrt{(e_{px} - e_{py})^2 + (e_{py} - e_{pz})^2 + (e_{pz} - e_{px})^2}, \qquad [3]$$

where e_{px}, e_{py}, e_{pz} are the three principal plastic strains. In the general biaxial-loading case, none of the principal plastic strains are zero, since the biaxial stresses can produce plastic deformation in the thickness direction. Although the constant appearing in front of the square root in Equation [3] is different than the one in Equation [2], both expressions result in effective values ($\bar{f}$, $\bar{e}_p$) for the uniaxial case which are equal to the actual values (f_o, e_{po}).

Marin, Ulrich, and Hughes (9), following a suggestion by L. W. Hu, equated expressions for the effective plastic strains for the uniaxial and general biaxial cases, respectively. The result was the following expression for the relationship between the plastic strain in the maximum-principal-stress direction x on a biaxially stressed specimen and its equivalent uniaxial plastic strain e_{po}:

$$e_{px}/e_{po} = (1 - 0.5B)/\sqrt{1 - B + B^2}. \qquad [4]$$

[5]Strictly speaking, the distortion-energy theory coincides with the von Mises criterion only for isotropic materials.

[6]It is improper to use the term deformation energy theory, since it does not distinguish between the total deformation energy and the distortion (shear) energy.

[7]Further discussion of the concepts of effective stress and effective plastic strain may be found in Reference (4). Also, it is noted that Equation [3] is based on the assumption that the plastic Poisson's ratio is 1/2. This assumption has been verified for the plastic portion of the strain in most metals by numerous investigators (8).

This relationship is depicted graphically in Figure 3 as a function of the biaxial ratio B.

In the determination of an offset yield strain equivalent to the uniaxial yield criterion e_{po} = 0.2 per cent, it is only necessary to enter Figure 3 with the appropriate B (or 1/B) ratio, move vertically to an intersection with the curve and horizontally to the e_{px}/e_{po} ordinate. This ordinate value when multiplied by e_{po} gives the equivalent offset strain. Except for the difference in offset strain, the procedure for determining the yield strength is the same as for the uniform plastic-strain criterion.

The equivalent plastic-strain criterion for biaxial yield strength has been used since 1951 at Penn State by Professor Marin and his associates in studies of many materials. It has also been used in recent work at Space Technology Laboratories (5).

The equivalent plastic-strain criterion is based partially on a particular theory of plasticity (the octahedral-shear-stress theory) and partially on actual test results for the material concerned. This is because the amount of offset strain used is based on the theory in conjunction with the biaxial ratio concerned, while the actual yield-stress values are taken from test results for the material, biaxial ratio, and offset.

To overcome the disadvantage suffered by the two previously described criteria, Dr. L. H. Lee of the University of Notre Dame has proposed the equivalent plastic-work criterion. This criterion is based on the work of Dr. D. C. Drucker (10). Basically, the concept here is simply to equate the plastic work done (strain energy) under biaxial loading to the plastic work done in straining uniaxially to a plastic strain of 0.2 per cent.

The unit plastic work done (measured in in-lb./cu. in. of volume of material) is equal to the total area beneath the stress-strain curve minus the elastic work, as shown schematically in Figure 4. Thus, in order to accurately determine plastic work from experimental data, it is necessary to planimeter an area from the stress-strain diagram.

Under biaxial loading, work is done in each of the biaxial principal directions. Thus, for each biaxial test, it is necessary to determine the area of the stress-strain curve for each of the two biaxial principal directions and then add these areas. Thus, experimental strain measurements in the second principal direction, which usually have not been reported in the literature, are required in order to utilize this criterion.

<u>Presentation and Reduction of Biaxial Yield-Strength Data</u>

In order to provide yield-strength values for a wide range of biaxial ratios, it is customary to present biaxial yield-strength results in the form of a biaxial yield-strength envelope. For information obtained from an internally pressured and axially loaded thin-walled cylinder, it is customary to have the hoop or circumferential stress f_C as the abscissa and the axial stress f_A as the ordinate. However, for flat sheet specimens, f_L and f_T representing the stresses in the longitudinal or rolling, and transverse or width directions, would be used instead of f_A and f_C, respectively.

Many different theories of multiaxial strength have been proposed for isotropic materials. Five of these have been discussed in detail by Marin (11) and can be represented, for the tension-tension quadrant of the biaxial-strength envelope, by the following nondimensional equations, where x denotes f_C/f_{ty} (and F_C/F_{ty}) and y is f_A/f_{ty} (and F_A/F_{ty}), where f_{ty} is the uniaxial tensile yield strength[8].

(1) Maximum-normal-stress theory, proposed by Rankine, best suited for so-called brittle materials:

$$(x-1)\ (y-1) = 0.$$

(2) Maximum-shear-stress theory, proposed by Coulomb, best suited for some ductile materials:

$$(x-1)\ (y-1) = 0.$$

(3) The octahedral-shear-stress theory, proposed by von Mises, best suited for many ductile materials:

$$x^2 + y^2 - xy = 1.$$

(4) Maximum-strain theory, proposed by Saint-Venant, not in current use:

$$(x-\mu y-1)\ (y-\mu x-1) = 0.$$

(5) Maximum total-strain-energy theory, in very limited use:

$$x^2 + y^2 - \mu xy = 1.$$

The nondimensional biaxial-yield-strength envelopes representing these five theories for tension-tension loading are given in Figure 5. However, there is often uncertainty in determining which of the theoretical envelopes corresponds best to actual test results for a given material under loadings corresponding to a range of biaxial-ratio values. Many times it is quite difficult to determine whether test data fall closer to the maximum-shear-stress theory or to the octahedral-shear-stress theory. Sometimes it appears that the data points form a smooth curve which differs from all of the theoretical envelopes. Also, for reasons of economy, it is customary to conduct biaxial tests for only a limited number of biaxial ratios (usually five: 0, 1/2, 1, 2, and ∞). This sometimes presents difficulty in fitting a smooth envelope curve to the data points. In order to overcome all of these difficulties, it is suggested here that a general conic curve, which is the most general second-degree algebraic curve, be used to reduce the data. This can be written as follows:

[8] As in MIL-HDBK-5, the symbol f denotes an actual or calculated stress, whereas the symbol F indicates a minimum or allowable strength.

$$ax^2 + by^2 + cxy + dx + ey = 1, \qquad [5]$$

where the coefficients a through e can be determined from test results for five different biaxial ratios. For an isotropic material, Equation [5] has a = b and d = |e|. Once the coefficients have been determined, Equation [5] can be used to compute points on the envelope curve corresponding to any intermediate biaxial ratio.

It is noted that each of the strength theories mentioned above can be represented by Equation [5] provided the coefficients are selected properly, as shown in Table 1.

TABLE 1. SPECIAL CASES OF THE GENERAL CONIC BIAXIAL-STRENGTH ENVELOPE

Strength Theory	Coefficients for Eq. [5] a	b	c	d	e
Maximum-normal-stress theory	0	0	-1	1	1
Maximum-shear-stress theory	0	0	-1	1	1
Octahedral-shear-stress theory	1	1	-1	0	0
Maximum-strain theory	μ	μ	$-(1+\mu^2)$	$1-\mu$	$1-\mu$
Maximum-total-strain-energy theory	1	1	$-\mu$	0	0
Hill's anisotropic plasticity equation	a*	b*	c*	0	0

*Arbitrary value.

For those who desire a more fundamental basis for the conic equation, reference is made to an equation given by Hill (12) for triaxial stress in an anisotropic material. First, the third normal stress and the two out-of-plane shear stresses in Hill's equation are set equal to zero since here we are dealing only with the biaxial case. Next, it is noted that when the biaxial normal-stress values used are principal-stress values, the in-plane shearing stress is zero. Furthermore, normal stresses appeared only as differences in Hill's equation (thus, d = e = 0 in Table 1), because he assumed that the superposition of a hydrostatic stress does not influence yielding. However, more recent experimental evidence obtained by Hu (13) suggests that hydrostatic stress can affect yielding significantly. Thus, the final result is the general conic equation suggested above.

Strictly speaking, the use of biaxial-stress values corresponding to biaxial plastic-strain values computed according to a particular hypothesis, such as Dorn and Thomsen's definition of effective plastic strain used in the equivalent plastic-strain yield criterion, is inconsistent with the above suggestion[9].

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[9] A more consistent procedure would be to determine the biaxial stresses for the equivalent plastic-strain values prescribed by Dorn and Thomsen's definition of effective plastic strain, next to use these stress values to determine the coefficients of the general conic curve, then to use these coefficients to define a new prescription for equivalent plastic strains, and finally to determine the stresses corresponding to these new plastic-strain values. This procedure could be repeated as many times as possible to obtain any desired accuracy. However, for the sake of standardization, it may be desirable to have the same offset for the same biaxial ratio regardless of the material. Since, in general, different materials would have different effective-stress coefficients, it is better to use the Dorn-Thomsen definition of effective plastic strain.

However, the error in using the Dorn-Thomsen hypothesis for most alloys of structural importance is believed to be quite small. Therefore, from a practical standpoint, it is believed that the use of the Dorn-Thomsen hypothesis in the equivalent plastic-strain criterion is justifiable.

Example

To illustrate the use of the conic-curve data-reduction method, it will be applied, in conjunction with the equivalent plastic-strain criterion, to biaxial data obtain by Goodman (5).

Table 2 lists the biaxial ratios used, the test conditions by which they were obtained, and the equivalent biaxial plastic strain in the maximum-principal-stress direction used in determining biaxial yield-strength values. It is to be noted that 0.173 is approximately 0.866 of 0.200 and that 0.100 is one-half of 0.200. Thus, the equivalent plastic-strain values used by Goodman are in agreement with Figure 3. Biaxial yield-strength data obtained on this basis are listed in Table 3. The yield-strength values were first reduced to nondimensional form, for ease of comparison, by dividing each by the uniaxial yield strength in tension. The material was found to be isotropic, within experimental error, and to have the same relative values regardless of strength level. That is, all strength levels showed yield strengths of about 1.00, 1.12, 1.00, 1.12 and 1.00 times the uniaxial f_{ty} at biaxial ratios of ∞, 2.0, 1.0, 0.5, and 0, respectively. For this reason, it was advantageous to combine all data; otherwise, it would have been necessary to work with data for each strength level separately.

TABLE 2. BIAXIAL RATIOS, TEST CONDITIONS, AND OFFSET YIELD-STRAIN VALUES USED BY GOODMAN (5)

Biaxial Ratio, B	Test Condition	Offset Strain in the Maximum-Principal-Stress Direction
∞	Uniaxial in the axial direction	0.200
2	Closed-end tube with internal pressure plus sufficient external axial tensile load(a)	0.173
1	Closed-end tube with internal pressure plus sufficient external axial tensile load(b)	0.100
0.5	Closed-end tube with internal pressure only	0.173
0	Uniaxial in the circumferential direction	0.200

(a) The internal pressure supplies an axial stress of pD/4t, where p is the pressure, D is the tube diameter, and t is the wall thickness. Thus, an external axial load equal to 3pD/4t must be added to obtain a total axial stress of pD/t which is twice the magnitude of the circumferential stress.

(b) External axial load must be equal to pD/4t.

TABLE 3. BIAXIAL YIELD-STRENGTH DATA FOR AISI 4340 STEEL(a)

Biaxial Ratio, B	Equivalent Yield Strength, ksi, for Indicated Tempering Temperature, F(b)						
	350	400	475	550	625	700	950
∞	-	224	225	226	219	-	163
	-	-	-	220	-	-	161
2	257	258	250	246	228	228	184
	250	250	253	236	248	224	175
	260	-	-	-	-	221	-
1	225	226	223	219	221	198	161
	230	225	223	219	220	200	161
	225	-	-	-	-	200	-
0.5	257	257	244	233	243	216	180
	245	256	250	233	240	221	172
	250	-	245	-	-	220	-
0	-	230	223	219	212	-	158
	-	-	-	-	215	-	-

(a) Thin-walled cylindrical specimens machined from bars. Data from (5).

(b) Specimens normalized at 1625 F for 10 minutes; air cooled, austenitized at 1525 for 10 minutes; oil quenched; tempered for 4 hours.

These data were then used to determine the values of the coefficients in Equation [5]. This equation is plotted in Figure 6 and represents the biaxial-yield-strength envelope for all data in Table 3, with ultimate tensile strengths ranging *from 180 to 260 ksi, inclusive.*

The final step taken was to expand this curve to several uniaxial f_{ty} levels to check the goodness of fit. This was done by multiplying the coordinates of various points of the curve by f_{ty}, in ksi. Expanded curves, along with plotted data points, are shown in Figure 7 for four tempering temperatures. As can be seen in Figure 7, agreement between specific test values and the curves was within usual experimental error.

Figure 6 represents the format recommended for the presentation of biaxial yield-strength data for hollow cylinders in MIL-HDBK-5; for sheet, f_L and f_T would be used in place of f_A and f_C. This format has many advantages for the designer; among them are:

- *It is dimensionless*, and thus can be used at various strength levels;
- It permits ready interpolation of data at intermediate biaxial ratios.

Summary

In this paper, the mechanical properties of thin metallic members under biaxial loading are discussed, with particular reference to practical use in the design of advanced aerospace vehicles.

The particular biaxial properties discussed include:

- Stress-strain curves
- Yield strength
- Ultimate strength.

Three criteria, for determining biaxial yield strength comparable to the 0.2 per cent offset strain criterion for uniaxial yield strength, are discussed. As an example, one of these criteria is applied, in conjunction with a generalized equation for describing the biaxial yield-strength envelope, to data for thin-walled cylinders of AISI 4340 steel heat treated to a wide range of strength levels.

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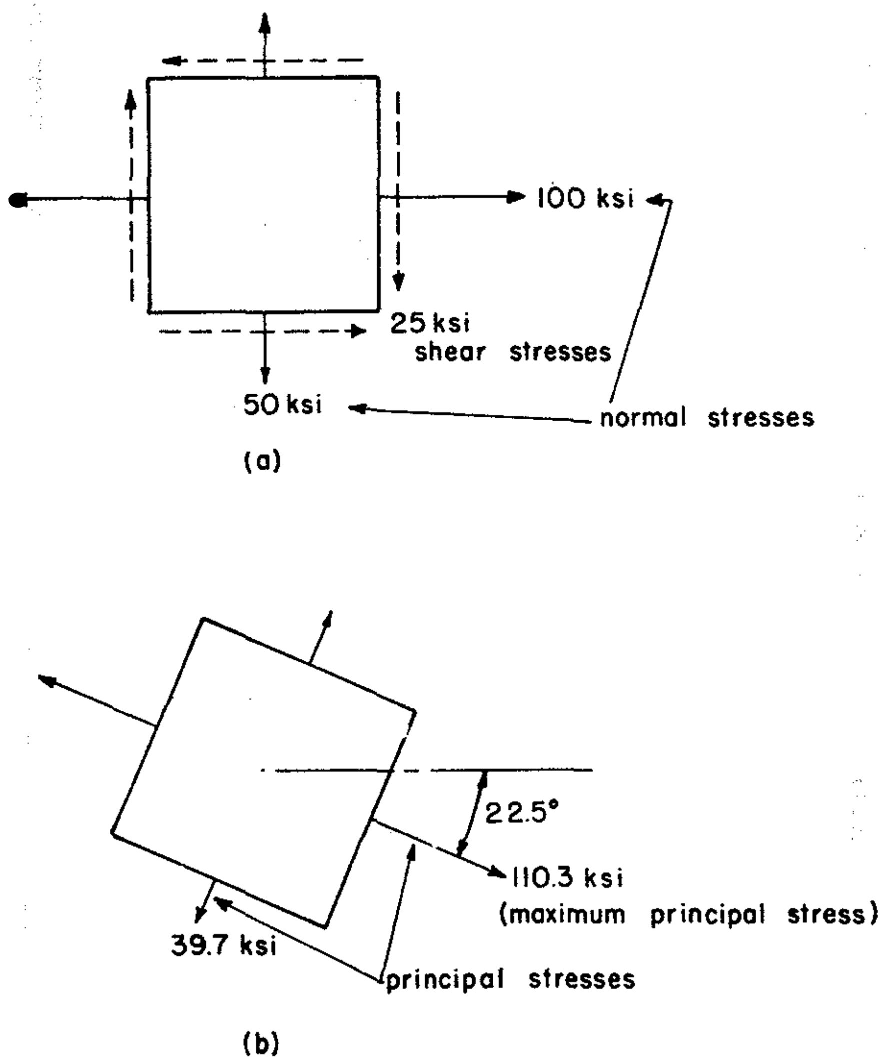

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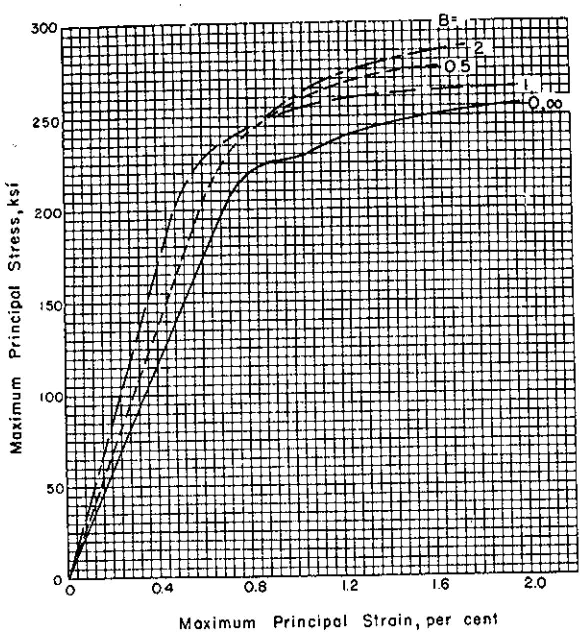
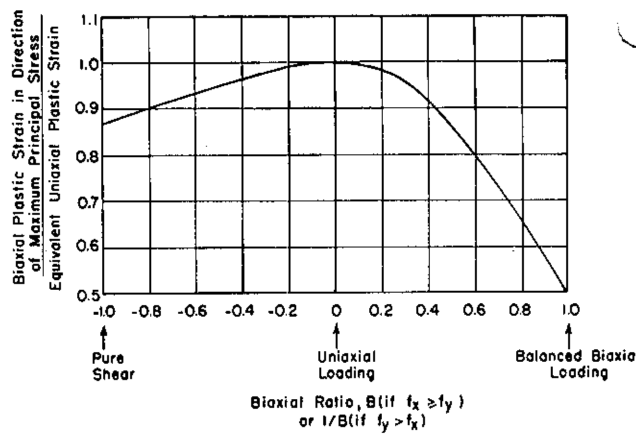
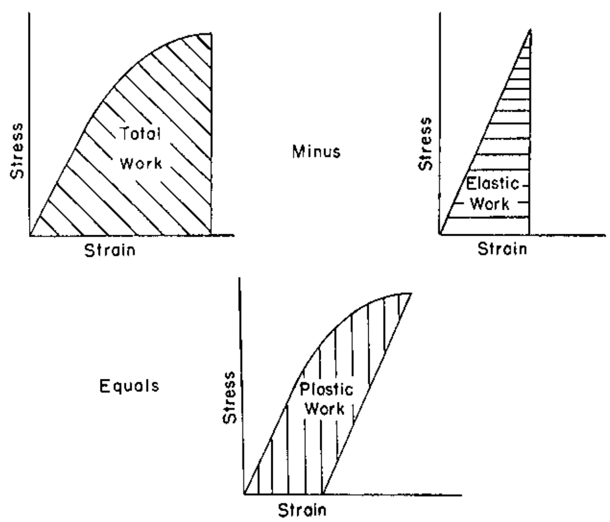


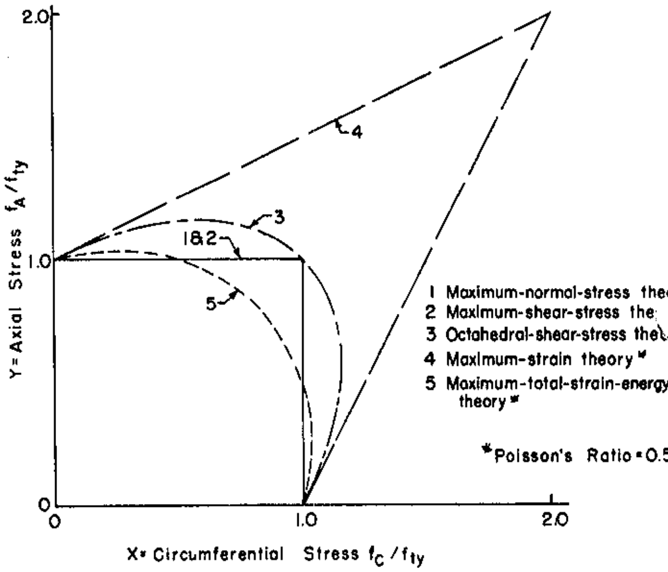
B=
2
0.5
1
0,∞
Maximum Principal Stress, ksi
Maximum Principal Strain, per cent



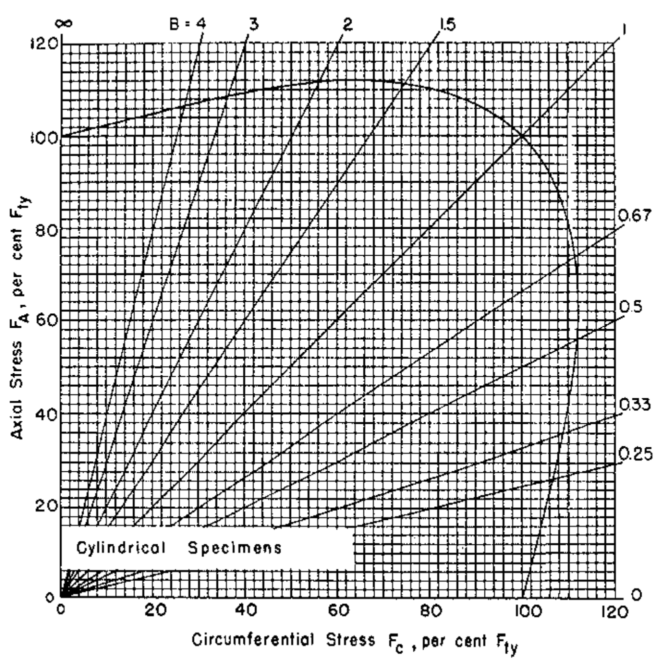
Biaxial Plastic Strain in Direction of Maximum Principal Stress
Equivalent Uniaxial Plastic Strain
Pure Shear
Uniaxial Loading
Balanced Biaxial Loading
Biaxial Ratio, B(if $f_x \geq f_y$) or 1/B(if $f_y > f_x$)



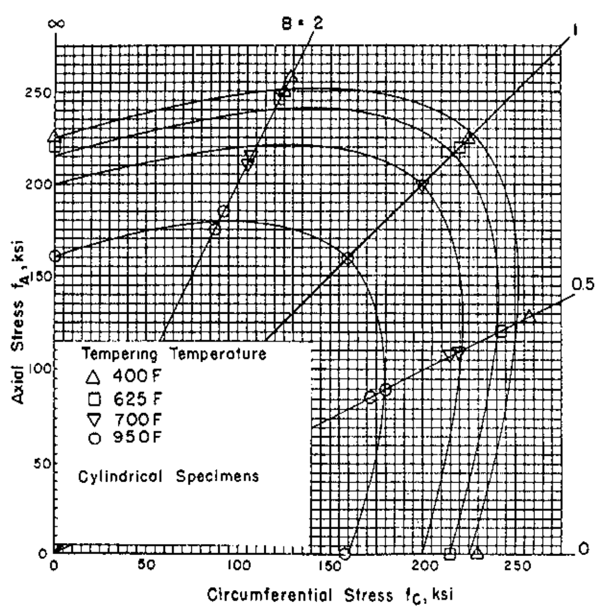
Stress
Total Work
Strain
Minus
Stress
Elastic Work
Strain
Equals
Stress
Plastic Work
Strain



Y = Axial Stress f_A/f_{ty}
X = Circumferential Stress f_C/f_{ty}
1&2
1 Maximum-normal-stress theory
2 Maximum-shear-stress theory
3 Octahedral-shear-stress theory
4 Maximum-strain theory*
5 Maximum-total-strain-energy theory*
*Poisson's Ratio = 0.5



B = 4
Axial Stress F_A, per cent F_{ty}
Cylindrical Specimens
Circumferential Stress F_c, per cent F_{ty}



B = 2
Axial Stress f_A, ksi
Tempering Temperature
△ 400 F
□ 625 F
▽ 700 F
○ 950 F
Cylindrical Specimens
Circumferential Stress f_C, ksi