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MINIMUM LUNAR ORBIT INCLINATION TO LUNAR EQUATORIAL PLANE FOR EARTH-LAUNCHED VEHICLE

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Summary

The minimum inclination (without a plane change) of the lunar orbit plane of an earth-launched vehicle to the lunar-equatorial plane is analyzed. Determination of this minimum inclination identifies those lunar orbit inclinations which cannot be attained without a plane change in the immediate vicinity of the moon. Launch azimuth restrictions are examined to establish launch windows; launch azimuth limits of 72 to 114 degrees were used. An investigation is included to show the launch window extensions that can be obtained by plane changes made in the earth parking orbit, while accelerating from parking orbit to injection velocity, or during the earth-moon transfer trajectory. The study results indicate, for the launch azimuth limits used, that the minimum lunar inclination varies between approximately plus or minus 15 degrees, that there are two launch windows per 24-hour period of four to six hours duration each, and that launch window extension via plane change is very expensive in terms of the required plane change velocity increment.

Introduction

In the planning of lunar missions, it is essential to know the orientation of the plane of the ballistic arrival trajectory with respect to the moon's surface. Certain missions require that a vehicle orbit be established in the equatorial plane of the moon. For this reason one of the parameters describing the orbit orientation, viz., the inclination of the arrival trajectory plane to the equatorial plane, is of prime importance since it indicates the relative cost of maneuvering from the arrival trajectory to the equatorial orbit. It is known from other lunar trajectory studies that a minimum inclination (without a plane change) to any given plane can be found.[1,2] The minimum inclination that can be attained ballistically depends upon earth-moon relative geometry. The relative configuration of the several pertinent earth and moon planes and the ecliptic is constantly changing so that the minimum inclination available varies with time. Lunar mission studies given in the literature have not dwelt upon the derivation of this inclination although Tolson has touched upon it.[2]

The purpose of this study is to determine this minimum inclination of the arrival, moon-referenced, trajectory plane with respect to the lunar-equatorial plane as a function of launch time, and to determine those launch dates or periods at which the minima occur. Launch windows are used to identify the data that is practicable. This data is presented to show its variation throughout lunar months and throughout the 18.6-year regression period of the lunar plane-ecliptic nodal line. The launch windows were established by range safety restrictions on launch azimuth at the Cape Canaveral launch site. Limits on launch azimuth of 72 to 114 degrees were used in the study.

Included in the study is an analysis of launch window extension by means of trajectory plane changes made in the earth parking orbit, at injection, or during transfer to the moon. The velocity increment needed to execute the plane change is used to present the data.

Discussion

Earth-Moon Geometry

As viewed from the earth along the earth-moon line, the orientation of the lunar-equatorial plane is continuously changing due to the orbital motion of the moon, as shown in Fig. 1. The "target plane", i.e., the lunar-equatorial plane, has an apparent rotary, wobbling motion with a period of one nodal lunar month so that the target plane is both wobbling and moving. An earth-moon-ecliptic geometry fact to be noted is that the nodal line of the lunar-equatorial plane on the lunar plane is parallel to the nodal line of the lunar plane on the ecliptic. The ascending node of one points in the direction of the descending node of the other.

The rather complicated earth-moon dynamic geometry dictates the coding of what amounts to an ephemeris of planar orientations and moon positions. The long period before a repetition of planar configurations indicates the practicality of a calendar type display of results. The large number of cases that must be computed to devise such a calendar demands an economical scheme of trajectory synthesis. Fortunately, the two-body conic section method described below provides such a process.

The simple form yielded by the two-body conic section method is well suited to this study.[3] Its accuracy has been demonstrated to be more than adequate.[1,4,5,6.] In this trajectory synthesis, an earth-referenced conic representing the transfer trajectory is determined to intersect the center of the moon, with the moon being considered as a massless point. The perigee altitude, the flight time from perigee to moon, and position of the moon at the time of arrival are specified parameters of the conic. The conic lies in a plane containing the center of the earth, the earth launch site, and the moon's center at the time of arrival. The moon's center is moving slowly compared to the motion of the launch site. Thus, the trajectory plane may be visualized as rotating about the earth-moon line so as to intersect the launch site in its daily motion, see Fig. 2. The fact that the target point would not be the moon's center, (except for direct hits) but would be a point offset from the center, is disregarded since the difference in cislunar trajectory inclination results is insignificant.

From the velocity vector of the conic at the moon's center is subtracted the velocity vector of the moon with respect to the earth. The resulting relative velocity vector is assumed to be the asymptotic vector of the arrival, moon-referenced hyperbola, see Fig. 3. (A similar scheme has been suggested by Egorov in his classical work.[7]) This asymptotic vector is essentially the velocity of the unperturbed trajectory relative to the center of the moon. This vector, directed through the moon's center, has been defined as the V_∞ vector. The plane of the hyperbola must contain this vector. This plane, which is the vehicle arrival, moon-referenced, trajectory plane, may be rotated about the V_∞ vector by small injection and/or midcourse corrections. The <u>least</u> inclination that the trajectory plane may take to <u>any</u> reference plane at the moon is the inclination of the V_∞ vector to that reference plane. It may take any steeper inclination however. The minimum inclination sought is simply the inclination of the V_∞ vector to the lunar-equatorial plane, see Fig. 4. It is illustrative to note that the intersection of this V_∞ vector with the lunar surface is the point of impact for a direct, vertical, lunar approach.

<u>Earth-Moon-Sun Model</u>

The moon was assumed to be in a mean circular orbit for the analysis. This approximation has little or no effect on the trends or magnitudes of the results; the only effect is a slight shift of the results along the time scale. All librations of the moon except the apparent wobbling of the lunar equator (mentioned earlier) were ignored.

Ecliptic longitude values of the target vector and the lunar plane-ecliptic nodal line at the time of lunar arrival were determined in terms of launch time from the respective expressions:

$$☾ = K_1 + 13.176397\left(d_L + \frac{h_L + t_T}{24}\right) \qquad (1)$$

$$\Omega = K_2 - 0.052953922\left(d_L + \frac{h_L + t_T}{24}\right) \qquad (2)$$

The ecliptic longitude of the earth-sun line at launch time was obtained from

$$A_S^* = K_3 + 0.98564734 d_L \qquad (3)$$

Values for the K_1, K_2 and K_3 coefficients depend upon the epoch used in preparing the data. In the analysis, it is assumed that the epoch selected begins at midnight, Greenwich Mean Time. Eqs. 1 through 3 were drawn from Reference 8; higher order terms were dropped as the additional accuracy would not appreciably affect the results.

<u>Analysis</u>

Only a brief analytic treatment is described here since the appendix contains the detailed derivation of the equations as do References 9, 10 and 11. Fig. 5 will be helpful in following the development.

<u>Minimum Lunar Inclination</u>. The ephemeris relationships of Eqs. 1 and 3 locate the target vector and the launch position vector. These two vectors establish the required vehicle trajectory plane. Orientation of the V vector within this plane was determined from two-body conic results which yielded the radial and azimuthal components of V at the moon with respect to the earth. Vector subtraction of the moon's velocity with respect to the earth leads to an expression for V_∞ which then permits the minimum lunar inclination to be defined. Having the trajectory plane located, the required launch azimuth can be determined. Limiting values on the launch azimuth establish launch windows which can be used to identify the data that is practicably useful.

The vehicle trajectory plane can be described by its unit normal I_V which can be obtained from the cross-product of the launch position and target vectors; succinctly,

$$I_V = \frac{I_L \times I_T}{\sin \phi} \qquad (4)$$

where the transfer angle ϕ follows from the dot product:

$$\phi = \cos^{-1}\left(I_L \cdot I_T\right) \qquad (5)$$

Since the launch azimuth vector lies in the trajectory plane, a second expression for the unit normal I_V can be formed as

$$I_V = I_L \times I_{Az} \qquad (6)$$

A comparison of the components of the expanded forms of Eqs. 4 and 6 leads to a formulation of launch azimuth:

$$A_z = \sin^{-1}\left[\frac{\cos\delta \sin(A - A_c)}{\sin\phi}\right] \qquad (7)$$

The inclination i_1 of the trajectory plane to the lunar plane can be obtained from the unit normals to these two planes by use of the equation

$$i_1 = \cos^{-1}\left(I_V \cdot I_M\right) \qquad (8)$$

Having the transfer trajectory plane-lunar plane inclination and the radial and azimuthal components of V with respect to the earth permits V_∞ to be referred to the moon in lunar plane coordinates by (using matrix notation)

$$V_{\infty \ell.p.} = \begin{pmatrix} V\sin\gamma \\ V\cos\gamma\cos i_1 - V_M \\ V\cos\gamma\sin i_1 \end{pmatrix} \quad (9)$$

The angle subtended by the V_∞ vector and the lunar-equatorial plane defines the minimum inclination - the desired study result. Using coordinate transformations to express V_∞ in a selenocentric, lunar-equatorial system and defining a unit V_∞ vector as I_∞, leads to a simple expression for this minimum inclination, viz.,

$$i_{VM} = \sin^{-1}\left[\left(I_{\infty \ell.eq.}\right)_z\right] \quad (10)$$

Trajectory Plane Change. The possibility of extending the launch window was examined by considering trajectory plane changes made in the earth parking orbit, during acceleration from parking orbit to transfer trajectory injection, and while in the transfer trajectory. A convenient parameter in describing the plane change is the velocity increment needed to execute the maneuver. The point of plane change was specified by the geocentric angle ϕ_1 subtended by the target vector and the vehicle position vector at the time of plane change. Fig. 6 is a sketch showing the launch trajectory plane (from launch to plane change) and the arrival trajectory plane (from plane change to lunar arrival).

For plane change in the earth parking orbit, the plane change angle i_{PC} can be related to the velocity increment ΔV by

$$i_{PC_P} = 2\sin^{-1}\left(\frac{\Delta V}{2V_C}\right) \quad (11)$$

The total velocity increment required to accelerate from circular parking orbital velocity to transfer trajectory injection velocity and simultaneously make a plane change can be obtained from

$$\Delta V_T = \sqrt{V_C^2 + V_{inj}^2 - 2V_C V_{inj}\cos i_{PC_{inj}}} \quad (12)$$

which with the definition

$$\Delta V = \Delta V_T - \left(V_{inj} - V_C\right) \quad (13)$$

leads to

$$i_{PC_{inj}} = \frac{|\Delta V|}{\Delta V}\cos^{-1}\left[1 - \frac{|\Delta V|}{V_C} + \frac{|\Delta V|}{V_{inj}} - \frac{(\Delta V)^2}{2V_C V_{inj}}\right] \quad (14)$$

where the sign of ΔV (as a computer program parameter) is used to determine the sign of the plane change angle. A change of plane during transfer trajectory can be written in terms of ΔV as

$$i_{PC_T} = 2\sin^{-1}\left(\frac{\Delta V}{2V_1\cos\gamma_1}\right) \quad (15)$$

Use of conic orbit relationships permits Eq. 15 to be rewritten as

$$i_{PC_T} = 2\sin^{-1}\left[\frac{\Delta V \cdot R_{inj} V_{inj}}{2\mu(1 + e\cos\eta_1)}\right] \quad (16)$$

The launch azimuth required when a plane change is to be made follows from a definition of the launch plane miss angle β. From the dot product of the normal to the launch plane and the target vector, the sine of this angle is

$$\sin\beta = I_V^* \cdot I_T \quad (17)$$

which leads to

$$\sin\beta = \sin A_Z^*\left[\sin\delta\cos\ell - \cos\delta\sin\ell\cos(A - A_C)\right] - \cos A_Z^*\cos\delta\sin(A - A_C) \quad (18)$$

An iterative solution of Eq. 18 yields the launch azimuth.

To get the unit normal to the arrival trajectory plane, use is made of the cross product of the unit position vector at the time of plane change and the target vector, viz.,

$$I_{V_{PC}} = \frac{I_{L_{PC}} \times I_T}{\sin\phi_1} \quad (19)$$

Having the arrival trajectory plane normal I_{VPC}, the remainder of the analysis with a plane change is identical to that of Eqs. 1 through 10 with one exception: I_V in the cited section should be replaced with I_{VPC} of Eq. 19 above.

Results

The equations presented above and in the appendix were coded for IBM 7090 digital computer solution. References 9, 10, and 11 contain the complete study data that have been obtained. Typical and summary results for an earth-moon transfer time of 60 hours are discussed herein.

Fig. 7 presents the transfer trajectory true anomaly of the target vector and the velocity components at the moon that were used in the computations; these are the two-body conic results mentioned in the Introduction. Launch azimuth limits of 72 to 114 degrees were used to establish exemplary launch windows for launch from Cape Canaveral, Florida.

Minimum Lunar Inclination

Presented below are the minimum inclinations for typical launch windows, a representative envelope of these minima showing their variation in a lunar month, and a plot showing the variation of certain maxima and minima of the minimum inclination envelope during the 18.6-year lunar regression period.

Fig. 8 shows a plot of launch azimuth for a typical 24-hour period during 13 June 1967. This figure indicates two launch windows, each of approximately five to six hours in duration, for the 24-hour period shown. It also presents the minimum lunar orbit inclination available without a plane change, as well as the required earth parking orbit coast angle associated with launch during this period. In computing the coast angle, a 15-degree value was assumed (based on other analyses) for each of the boost angles - launch-to-parking-orbit and parking-orbit-to-injection (the respective

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angles ρ_1 and ρ_2 of Eq. A28). The cyclic variation of the minimum inclination during a 24-hour period is caused by the plane of the earth-referenced trajectory rotating about the earth-moon target line as the launch site moves with the earth's rotation. Since the inclination of the moon-referenced trajectory to the lunar-equatorial plane depends upon the orientation of the earth-referenced trajectory plane, the variation of minimum inclination with hour-of-the-day is reasonable.

The curves of Fig. 9, for a one-week period, show how envelopes can be formed for the coast-angle requirements and minimum lunar inclination. These envelopes provide a convenient means to examine the cycles for various periods of interest in earth-moon geometry. Coast angle and minimum inclination angles are shown in Fig. 10 for a 60-day period; this plot illustrates the periodicity of these angles with respect to a lunar month. The one-week period of Fig. 9 is indicated on Fig. 10 by the shading. Fig. 10 shows, for the 60-day period considered, that the minimum lunar orbit inclination to the lunar-equatorial plane varies between approximately plus or minus twelve degrees within the launch window specified. It should be remembered that only discrete line segments within the envelopes are valid data as was shown in Fig. 9; furthermore, the minimum inclination envelope contains the segments for both daily launch windows while the coast angle envelopes each contain only one of the two daily segments. The cyclic variation of the minimum inclination during a lunar month is due to two factors. Throughout a lunar month the earth-moon target line varies with respect to the earth-equatorial plane - the monthly declination cycle. This target line is also varying with respect to the moon-equatorial plane. To gain some insight into the variation of the minimum inclination during a lunar month, consider Fig. 1 and the (projected) V_∞ vector shown at two diametrically opposite positions in the moon's orbit about the earth. On Fig. 1 the two projections of V_∞ (onto the plane normal to the lunar plane and containing the earth-moon line) are assumed fixed and inclined to the lunar plane at the angle α. With this premise, the cited figure illustrates, qualitatively, how the minimum inclination takes a cyclic variation during a lunar month.

The particular earth-moon-ecliptic configuration of Fig. 1 depicts roughly conditions that will exist in 1978. At that time, the node of the lunar plane on the ecliptic coincides with its node on the earth-equatorial plane and both coincide with the Vernal Equinox. The node of the lunar-equatorial plane on the lunar plane is parallel to the Vernal Equinox, at that time, and the lunar plane is at its least inclination to the earth-equatorial plane.

An entire 18.6-year period was investigated to cover all possible relative positions of the earth and moon. The 18.6-year time period encompasses one regression cycle of the lunar plane-ecliptic nodal line. Sixty-day periods separated by 365 days were considered to get a complete sampling of data. The results obtained were quite similar to those shown in Figs. 8 through 10. The excursions of certain maxima and minima for the minimum lunar inclination data are shown in Fig. 11 for a typical 18.6-year period; the inset on this figure identifies the maximum and minimum points under consideration. Fig. 11 indicates that the minimum inclination envelopes tend to flatten during the two or three-year period leading to the minimum lunar declination which occurs in 1978. This flattening is perceivable since the variation in lunar declination within a lunar month is smaller during this period.[12,13] As a help in understanding the regressional-period variation of the minimum inclination, consider the moon to be fixed in its orbit. Then the nodal line of the lunar-equatorial plane on the lunar plane is slowly rotating clockwise, with the regressional period. The declination of the moon is varying with the same period (unless it is zero) since the inclination of the lunar plane to the earth-equatorial plane is changing. These latter two variations are superimposed on all others and account for the need, as filled by this study, of an automatic digital computing machine program and the covering of at least an 18.6-year period.

Launch Window Extension

Results showing the effect of trajectory plane change on launch window are presented in Figs. 12 and 13. The data of Fig. 12 shows the change in launch azimuth for a plane change made at ϕ_1 = 20 degrees using velocity increment values of plus or minus 500 fps. The launch window extension Δt that results from this plane change is identified on the figure. It was found that Δt was nearly linear with ΔV for ΔV and ϕ_1 values of interest, viz.,

$$|\Delta V| \leq 500 \text{ fps}$$
$$\phi_1 = 2 \text{ to } 270 \text{ degrees}$$

Fig. 13 is a summary plot of the Δt - ΔV - ϕ_1 relationship for a transfer time of 60 hours. This figure shows that the launch window can be most effectively extended by plane changes made at ϕ_1 values of approximately 15 degrees. The earth radial distance indication on the curve demonstrates that this corresponds to about three-tenths of the distance to the moon which is in the same general region where midcourse corrections are likely to be made. The shape of the curve of Fig. 13 can be explained by the following: From a geometric standpoint, plane changes at ϕ_1 = 90 or 270 degrees are the most effective, while plane changes for ϕ_1 values near zero or 180 degrees are quite ineffective. However, at ϕ_1 = 270 degrees the vehicle is in an earth parking orbit with the associated circular orbital velocity; this high circular velocity offsets the advantages of the geometric effectiveness. At ϕ_1 = 90 degrees, the vehicle, on the transfer trajectory, still has a high velocity. Although the plane change can be combined with the acceleration to injection at ϕ_1 values near 180 degrees, the geometric disadvantage in this vicinity more than offsets the benefit of the combination. The study data of Fig. 13 indicate that the lower velocity on the transfer trajectory and the geometric effectiveness combine to yield optimal results for a ϕ_1 value near 15 degrees.

Conclusions

Study results have been presented above, for an entire 18.6-year lunar regressional period, illustrating the minimum lunar inclination (without a plane change) and the associated earth parking orbit coast angle. Study premises included an earth-moon transfer time of 60 hours and launch azimuth restrictions of 72 to 114 degrees for the Cape Canaveral launch site. Additional results were presented showing the launch window extensions available via trajectory plane change. These results permit the following conclusions to be drawn:

1. There are two launch windows per 24-hour period, each of four to six hours in duration.

2. The minimum lunar inclination varies between approximately plus or minus 15 degrees.

3. There are at least two times during each lunar month when a near lunar-equatorial orbit can be obtained without a plane change.

4. Generally there is at least one direct ascent possibility during each lunar month (with a few possible exceptions). There may be up to five or six direct ascents possible within a lunar month.

5. Launch window extension via trajectory plane change is quite expensive in terms of the required velocity increment.

Further study efforts for other earth-moon transfer times up to 95 hours have revealed data trends and magnitudes quite similar to those reported herein.

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Nomenclature

☾	Ecliptic longitude of target vector at lunar arrival, measured from equinox of date, see Fig. 5
A	Right ascension of target vector at arrival
A_C	Right ascension of launch site at launch
A_G	Right ascension of Greenwich at launch
A_{PC}	Right ascension of point of trajectory plane change
A_S	Right ascension of mean sun at beginning of launch day
A_S^*	Ecliptic longitude of mean sun at beginning of launch day, measured from equinox of date
A_Z	Launch azimuth measured east from north
A_Z^*	Launch azimuth for case when trajectory plane change is to be made
b	Angle subtended by target vector and ecliptic
d_L	Number of days from the epoch to launch time
e	Earth-Moon transfer orbit eccentricity
h_L	Number of hours from the beginning of the launch day to launch time, Greenwich Mean Time
i	Inclination of the lunar plane to the ecliptic
i_{EL}	Inclination of the lunar-equatorial plane to the lunar plane
$i_{PC_{inj}}$	Angle of trajectory plane change made while accelerating from parking orbit to injection velocity
i_{PC_P}	Angle of trajectory plane change made in parking orbit
i_{PC_T}	Angle of trajectory plane change made while in transfer trajectory
i_V	Inclination of the vehicle trajectory plane to the earth-equatorial plane
i_{VM}	Angle subtended by the V_∞ vector and the lunar-equatorial plane
i_1	Inclination of the vehicle trajectory plane to the lunar plane

I_{A_Z} Geocentric unit vector parallel to the launch azimuth vector

I_L Geocentric unit vector directed to the launch site at launch

$I_{L_{PC}}$ Geocentric unit position vector to point of trajectory plane change

I_M Geocentric unit vector normal to the lunar plane

I_T Geocentric unit target vector

I_V Geocentric unit vector normal to the vehicle trajectory plane

I_V^* Geocentric unit normal to arrival trajectory plane, i.e., after plane change

$I_{V_{PC}}$ Geocentric unit normal to launch trajectory plane, i.e., prior to plane change

I_Z Unit vector normal to lunar-equatorial plane

I_∞ Unit vector of V_∞

I_Ω Geocentric unit vector along the ascending node of the lunar plane on the ecliptic

K_1, K_2, K_3 Constant terms of ephemeris equations, see Eqs. 1, 2, 3, respectively

ℓ Latitude of launch site

ℓ_{PC} Declination of point of plane change

R_{inj} Geocentric radius to transfer trajectory injection point.

Δt Launch window extension time increment, see Fig. 12.

t_T Cislunar transfer time in hours

V Velocity of the vehicle at the moon with respect to the earth

V_C Earth parking orbit circular velocity

V_{inj} Transfer trajectory injection velocity

V_M Velocity of the moon with respect to the earth

V_1 Local vehicle velocity on the transfer trajectory

V_∞ Approach asymptotic velocity vector with respect to the moon

ΔV Velocity increment needed to execute trajectory plane change

ΔV_T Velocity increment needed to simultaneously accelerate from parking orbit velocity to injection velocity and execute a trajectory plane change

Greek Symbols

α An arbitrary parameter used on Fig. 1

β Angle subtended by the target vector and the launch trajectory plane

δ Declination of target vector

ϵ Inclination of earth-equatorial plane to the ecliptic

η True anomaly of the cislunar transfer trajectory at lunar arrival

η_1 True anomaly of plane change point (for the case of plane change in the transfer trajectory)

γ Flight path angle of the V vector with respect to the local earth horizontal

γ_1 Local transfer trajectory flight path angle referred to the local earth horizontal

μ Earth gravitational field constant

Ω Angle in the ecliptic plane eastward from the Vernal Equinox to the ascending node of the lunar plane on the ecliptic, see Figure 1

Ω_{EL} Angle in the lunar plane counterclockwise from a target vector extension to the ascending node of the lunar-equatorial plane on the lunar plane, see Fig. A6

Ω_L Longitude eastward from Greenwich to the launch site

ϕ Geocentric angle subtended by launch position and target vectors

ϕ_{PC} Geocentric angle subtended by launch and plane change position vectors

ϕ_1 Geocentric angle subtended by the plane change position vector and the target vector

ψ Geocentric angle subtended by launch position vector and line of apsides of transfer trajectory

ρ_1 Earth central angle traversed by vehicle during boost from launch to earth parking orbit

ρ_2 Earth central angle traversed by vehicle during acceleration from parking orbit velocity to transfer trajectory injection velocity

θ Earth parking orbit coast angle

Subscripts

ec Geocentric ecliptic coordinate system

eq Geocentric earth-equatorial coordinate system

ℓ.eq. Selenocentric lunar-equatorial coordinate system, x axis along the ascending node of the lunar-equatorial plane on the lunar plane and z axis coincident with the lunar north pole

ℓ.p. Selenocentric lunar plane coordinate system, x axis outward from earth along the target vector and z axis northward

z z component of a vector

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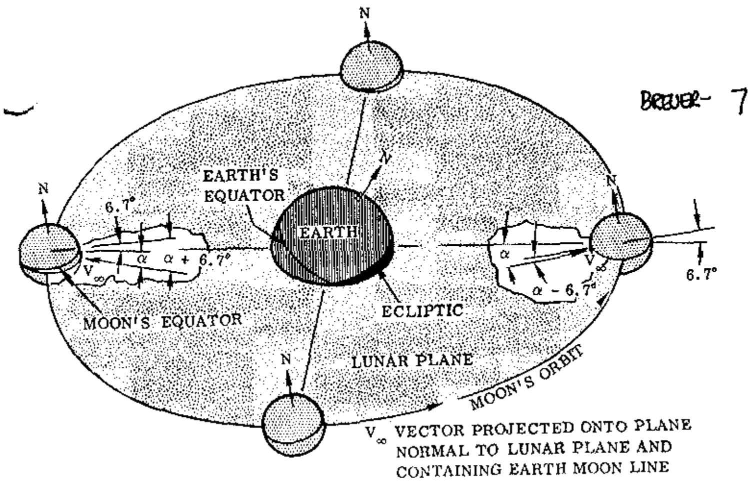

Figure 1. Typical Earth-Moon Geometry

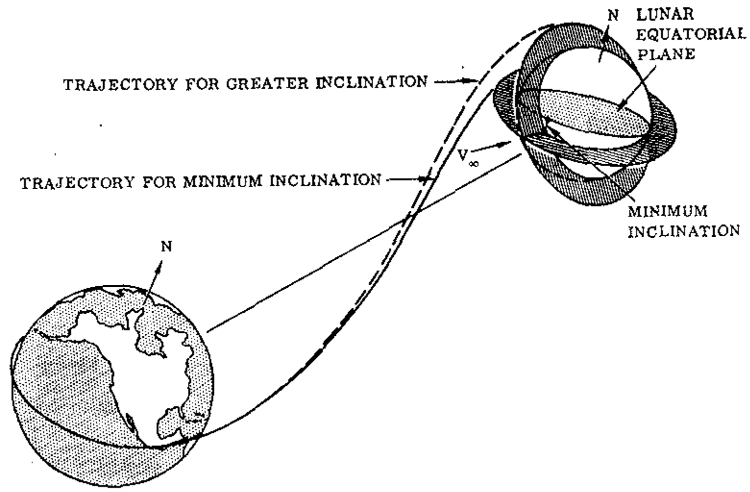

Figure 2. Vehicle Lunar-Orbit Inclination

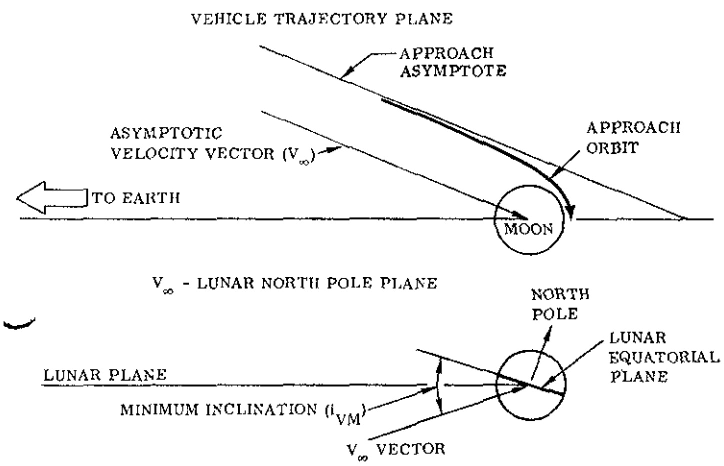

Figure 3. Lunar Approach Geometry

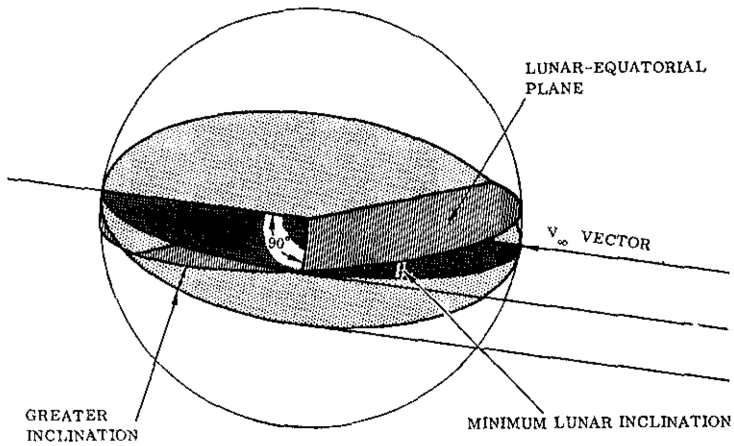

Figure 4. Inclination of Vehicle Orbit to Lunar-Equatorial Plane

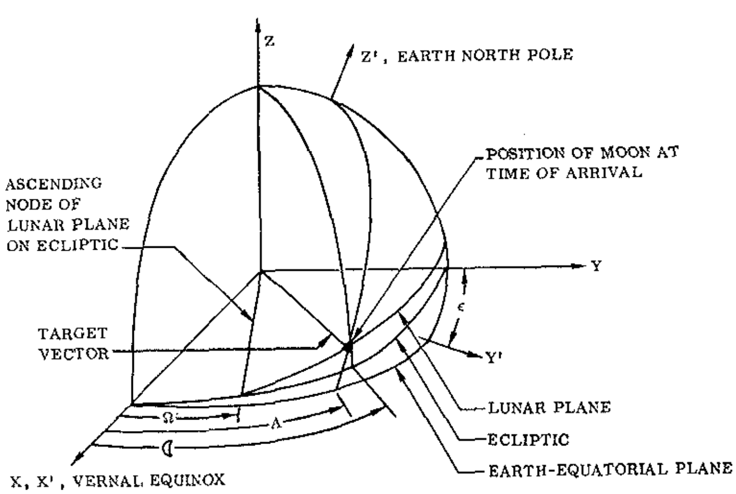

Figure 5. Geocentric Earth, Moon and Ecliptic Geometry

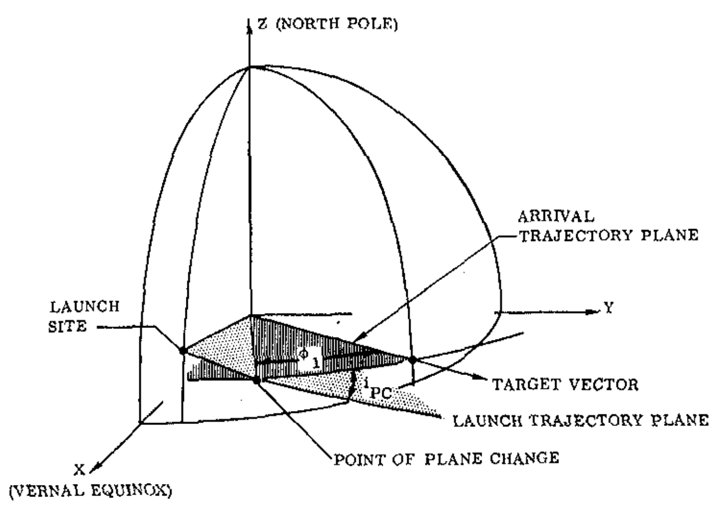

Figure 6. Trajectory Plane Change Geometry

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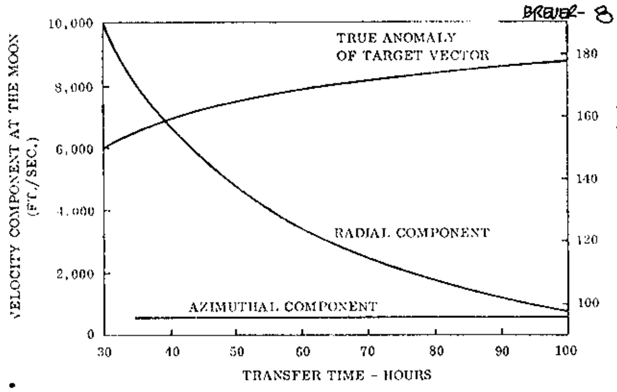

Figure 7. Two-Body Conic Section Results

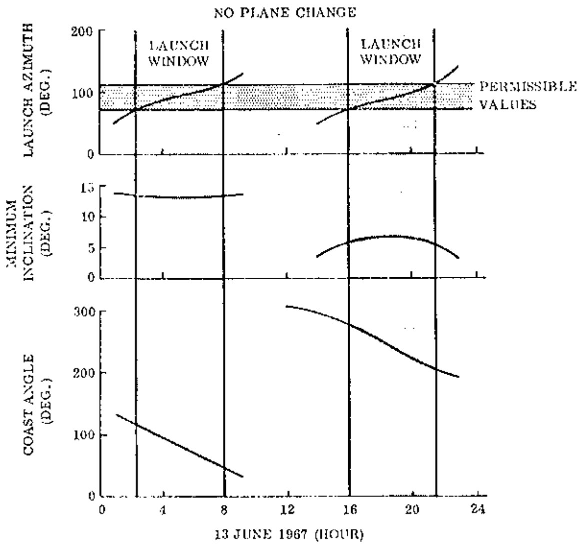

Figure 8. Earth-Moon Launch Window, Minimum Lunar Inclination and Earth Parking Orbit Coast Angle

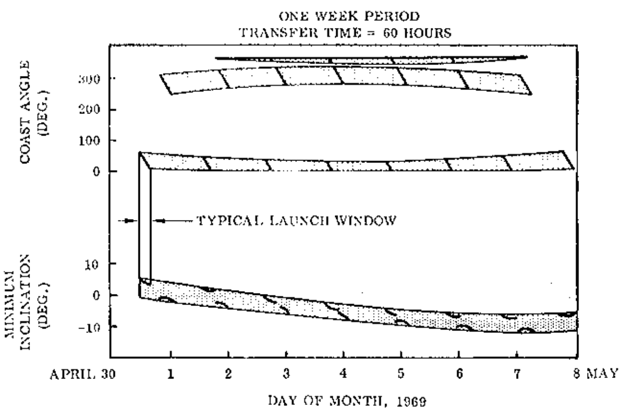

Figure 9. Earth Parking Orbit Coast Angle and Minimum Lunar Inclination

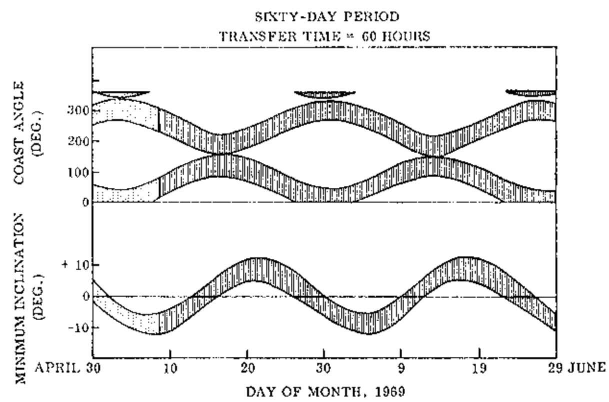

Figure 10. Earth Parking Orbit Coast Angle and Minimum Lunar Inclination

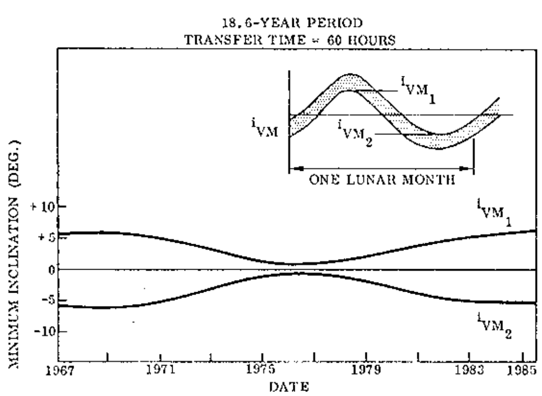

Figure 11. Excursions of Certain Maxima and Minima of Minimum Lunar Inclination

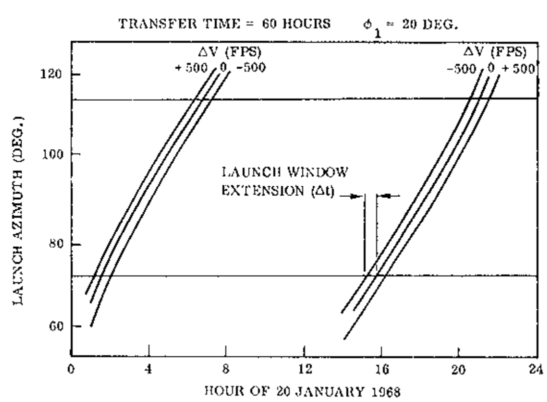

Figure 12. Effect of Plane Change Velocity Increment on Launch Azimuth

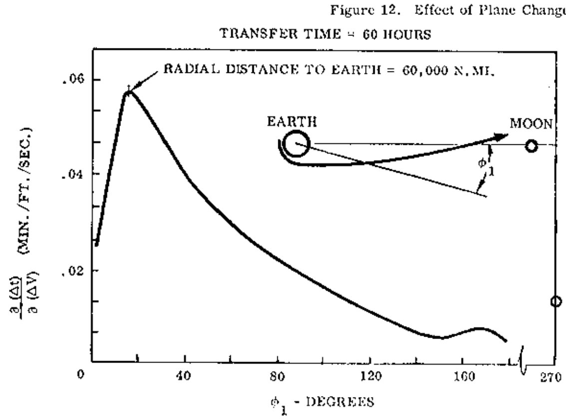

Figure 13. Launch Window Extension

Appendix

This appendix presents a detailed derivation of the equations and terms necessary to evaluate the summary expressions given in the main body of the paper. Matrix notation will be used where appropriate.

A geocentric unit target vector can be written in the ecliptic coordinate system directly from Fig. 5 as

$$I_{T_{ec}} = \begin{pmatrix} \cos (\cos b \\ \sin (\sin b \\ \sin b \end{pmatrix} \qquad (A1)$$

The angle b subtended by the target vector and the ecliptic is given by a right spherical trigonometric relationship:

$$b = \sin^{-1}\left[\sin i \sin((- \Omega)\right] \qquad (A2)$$

The angle b will be a first or fourth quadrant angle accordingly as the inverse sine argument of Eq. A2 is positive or negative; the lunar plane-ecliptic inclination i is assumed always positive. A coordinate rotation about the x axis through the angle ϵ can be used to express the target vector in the earth-equatorial system as

$$I_{T_{eq}} = \begin{pmatrix} \cos (\cos b \\ \cos \epsilon \sin (\cos b - \sin \epsilon \sin b \\ \sin \epsilon \sin (\cos b + \cos \epsilon \sin b \end{pmatrix} (A3)$$

A second expression for the target vector can be written directly from Fig. 5:

$$I_{T_{eq}} = \begin{pmatrix} \cos A \cos \delta \\ \sin A \cos \delta \\ \sin \delta \end{pmatrix} \qquad (A4)$$

Equating the x components of Eqs. A3 and A4 gives

$$\delta = \cos^{-1}\left(\frac{\cos (\cos b}{\cos A}\right) \qquad (A5)$$

and equating the y/x quotients yields

$$A = \tan^{-1}\left(\frac{\cos \epsilon \sin (\cos b - \sin \epsilon \sin b}{\cos (\cos b}\right) \qquad (A6)$$

From Fig. 5, it can be noted that A is a first or fourth quadrant angle when (is in the first or fourth, and is a second or third quadrant angle when (is in the second or third; A may not be in the same quadrant as (, however. The quadrant of δ is first or fourth as the sign of

$$\sin \delta = \sin \epsilon \sin (\cos b + \cos \epsilon \sin b \qquad (A7)$$

is plus or minus; Eq. A7 results from equating the z components of Eqs. A3 and A4.

The launch position vector can be analytically defined by finding the right ascension of the mean sun. Using Eq. 3 and Fig. A1, this is

$$A_S = \tan^{-1}\left(\tan A_S^* \cos \epsilon\right) \qquad (A8)$$

where A_S and A^*_S are always in the same quadrant. Since the mean sun of Eq. 3 is midnight with respect to Greenwich, the right ascension of Greenwich at launch is

$$A_G = A_S + 180 + 15 h_L \qquad (A9)$$

so that the launch site right ascension follows as:

$$A_C = A_G + \Omega_L \qquad (A10)$$

An expression for the launch position vector can now be written directly from Fig. A2 as

$$I_{L_{eq}} = \begin{pmatrix} \cos l \cos A_C \\ \cos l \sin A_C \\ \sin l \end{pmatrix} \qquad (A11)$$

The vehicle trajectory plane normal relationship of Eq. 4 can be expanded using Eqs. A11 and A3 to yield

$$I_V = \frac{1}{\sin \phi}\begin{pmatrix} \sin A_C \cos l \sin \delta - \sin A \sin l \cos \delta \\ \cos A \sin l \cos \delta - \cos A_C \cos l \sin \delta \\ \cos l \cos \delta \sin(A - A_C) \end{pmatrix} \qquad (A12)$$

where ϕ can be obtained from Eqs. 5, A11 and A3 as the form:

$$\phi = \cos^{-1}\left[\cos l \cos \delta \cos(A_C - A) + \sin l \sin \delta\right] \qquad (A13)$$

Referring to Fig. A3, and recalling the eastward launch assumption, leads to the quadrant determination of ϕ. If the z component of Eq. A12 is positive, ϕ is a first or second quadrant angle whereas if this z component is negative, ϕ is in the third or fourth.

Eq. 6 can be defined in detail by the following: A geocentric unit vector parallel to the launch azimuth vector can be written directly from Fig. A4 in the double-primed system as

$$I''_{A_Z} = \begin{pmatrix} 0 \\ \sin A_Z \\ \cos A_Z \end{pmatrix} \qquad (A14)$$

A rotation of x" about y" from the launch site to the earth-equatorial plane followed by a rotation of x', about y' from the launch-site right ascension to the Vernal Equinox leads to

$$I_{A_Z} = \begin{pmatrix} -\cos A_C \cos A_Z \sin l - \sin A_Z \sin A_C \\ -\sin A_C \cos A_Z \sin l + \sin A_Z \cos A_C \\ \cos A_Z \cos l \end{pmatrix} \qquad (A15)$$

Substitution of Eqs. A11 and A15 into 6 gives the result:

$$I_V = \begin{pmatrix} \sin A_C \cos A_Z - \cos A_C \sin l \sin A_Z \\ -\cos A_C \cos A_Z - \sin A_C \sin l \sin A_Z \\ \cos l \sin A_Z \end{pmatrix} \quad (A16)$$

Determination of the launch azimuth (given by Eq. 7) quadrant is possible by establishing the sign of cos A_Z. Equating the x components of Eqs. A12 and A16 yields

$$\cos A_Z = \frac{\sin A_C \cos l \sin \delta - \sin A \sin l \cos \delta}{\sin \phi \sin A_C} + \frac{\cos A_C \sin l \sin A}{\sin A_C} \quad (A17)$$

where sin A_Z can be obtained from Eq. 7.

Expansion of Eq. 8 for the inclination of the trajectory plane to the lunar plane leads to a lengthy expression for which no convenient analytic simplification was found. Consequently, in using this equation, the vector components were numerically evaluated prior to forming the dot product. The lunar plane unit normal I_M can be found from the cross product:

$$I_M = \frac{I_\Omega \times I_T}{\sin(☾ - \Omega)} \quad (A18)$$

From Fig. 5, in the ecliptic system,

$$I_{\Omega_{ec}} = \begin{pmatrix} \cos\Omega \\ \sin\Omega \\ 0 \end{pmatrix} \quad (A19)$$

Conversion of Eq. A19 to the earth-equatorial system and substitution into Eq. A18 together with Eq. A3 results in

$$I_{M_{eq}} = \frac{1}{\sin(☾-\Omega)} \begin{pmatrix} \sin\Omega \sin b \\ -\cos\epsilon \cos\Omega \sin b + \sin\epsilon \cos b \sin(\Omega - ☾) \\ -\sin\epsilon \cos\Omega \sin b - \cos\epsilon \cos b \sin(\Omega - ☾) \end{pmatrix} \quad (A20)$$

The quadrant of i_1 (of Eq. 8) is needed to indicate whether the vehicle, in arriving at the moon, approaches the lunar plane from above or below. Recalling the eastward launch assumption and using Fig. A5, define a first or fourth quadrant i_1 to indicate, respectively, that the vehicle approaches the lunar plane from below or above. Then, since I_T is positive along the positive x axis of Fig. A5 (out of the plane of the paper toward the reader), and since the cross-product $I_M \times I_V$ lies along the x axis (but not necessarily in the same direction), it can be concluded that

$$i_1 \text{ has the sign of } \left[(I_M \times I_V) \cdot I_T\right] \quad (A21)$$

The unit vector I_∞ is defined by

$$I_\infty = \frac{V_\infty}{|V_\infty|} \quad (A22)$$

Expressing I_∞ in a selenocentric, lunar-equatorial system requires two coordinate rotations. Use is made of Fig. A6 and of the fact that the ascending node of the lunar-equatorial plane on the lunar plane is parallel to the ascending node of the lunar plane on the ecliptic. The first rotation is of x about z from its target vector orientation to the lunar-equatorial plane ascending node giving the x'-y'-z' system of Fig. A6. Then, a rotation of y' about x' from the lunar plane to the lunar-equatorial plane makes z' coincident with the lunar north pole. Noting that

$$\Omega_{EL} = 180 - ☾ + \Omega \quad (A23)$$

the desired form of the unit vector I_∞ is

$$I_{\infty l_{eq}} = \begin{pmatrix} \cos\Omega_{EL} & \sin\Omega_{EL} & 0 \\ -\sin\Omega_{EL}\cos i_{EL} & \cos\Omega_{EL}\cos i_{EL} & \sin i_{EL} \\ \sin\Omega_{EL}\sin i_{EL} & -\cos\Omega_{EL}\sin i_{EL} & \cos i_{EL} \end{pmatrix} \frac{V_{\infty l.p.}}{|V_{\infty l.p.}|} \quad (A24)$$

Further analytic expansion of Eq. A24 is not convenient; accordingly, numerical evaluation was used to obtain the z component required by Eq. 10. The expression of Eq. 10 follows directly from

$$\sin i_{VM} = \cos(90 - i_{VM}) = (I_{\infty l.eq.}) \cdot (I_{Zl.eq.}) = (I_{\infty l_{eq}})_z \quad (A25)$$

since

$$I_{Zl.eq.} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (A26)$$

The minimum lunar orbit inclination i_{VM} is a first or fourth quadrant angle accordingly as Eq. A25 is positive or negative.

In determining the earth parking orbit coast angle, the perigee of the earth-moon trajectory was assumed as a nominal injection point. From Fig. A3 the angle ψ from launch to the injection point is

$$\psi = \phi - \eta \quad (A27)$$

where the transfer trajectory true anomaly η is taken from the two-body conic results cited earlier. Then, from Fig. A7, the earth parking orbit coast angle θ can be written directly as

$$\theta = \psi - (\rho_1 + \rho_2) \quad (A28)$$

The angles ρ_1 and ρ_2 are, respectively, the boost angles from launch to parking orbit and from parking orbit to injection. Since the coast angle cannot be negative, any negative value from Eq. A28 must have 360 degrees added to it.

Eqs. 11 through 16 adequately explain the relationships between the plane change velocity increment and the plane change angle.

The angle β of Eq. 17 can be obtained by using a spherical trigonometric relationship (and Fig. 5) to give

$$\beta = \sin^{-1}\left(\sin i_{PC} \sin \phi_1\right) \qquad (A29)$$

The launch plane unit normal $I_V{}^*$ of Eq. 17 is given by the equation for I_V, viz., Eq. A16 while the unit target vector is given by Eq. A4. Substitution of Eqs. A4 and A16 into Eq. 17 yields Eq. 18.

The unit position vector at the time of plane change ($I_{L_{PC}}$), see Eq. 19, can be defined in terms of the declination and right ascension of the plane change point. Using spherical trigonometric relationships the declination is

$$l_{PC} = \sin^{-1}\left(\sin l \cos \phi_{PC} + \cos l \cos A_Z^* \sin \phi_{PC}\right) \quad (A30)$$

while the right ascension can be obtained from

$$\sin\left(A_{PC} - A_C\right) = \frac{\sin \phi_{PC} \sin A_Z^*}{\cos l_{PC}} \quad (A31)$$

and

$$\cos\left(A_{PC} - A_C\right) = \frac{\cos \phi_{PC} - \sin l_{PC} \sin l}{\cos l_{PC} \cos l} \quad (A32)$$

Both Eqs. A31 and A32 are needed for quadrant determination of A_{PC}. Having l_{PC} and A_{PC}, the plane change unit position vector can be written directly from Fig. 6 as

$$I_{L_{PC}} = \begin{pmatrix} \cos A_{PC} \cos l_{PC} \\ \sin A_{PC} \cos l_{PC} \\ \sin l_{PC} \end{pmatrix} \quad (A33)$$

Substitution of Eqs. A4 and A33 into Eq. 19 leads to a formulation of the normal to the arrival trajectory plane:

$$I_{V_{PC}} = \frac{1}{\sin \phi_1} \begin{pmatrix} \sin A_{PC} \cos l_{PC} \sin \delta - \sin A \sin l_{PC} \cos \delta \\ \sin l_{PC} \cos A \cos \delta - \cos A_{PC} \cos l_{PC} \sin \delta \\ \cos l_{PC} \cos \delta \sin (A - A_{PC}) \end{pmatrix} \quad (A34)$$

This completes the detailed derivation needed in the analysis.

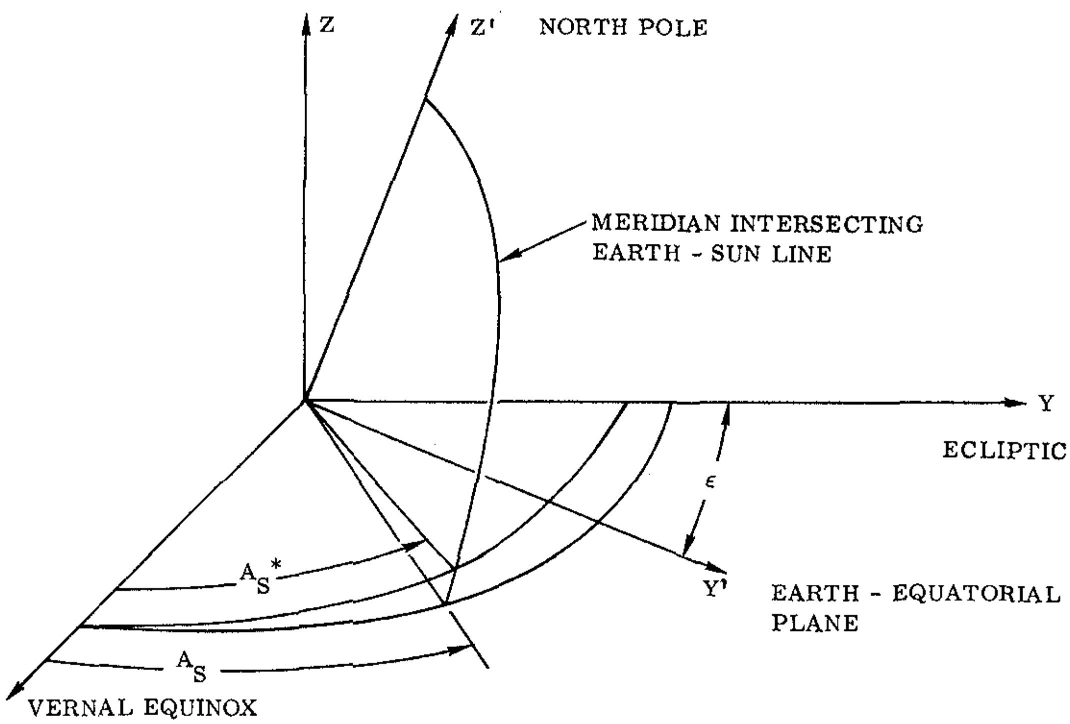

Figure A1. Right Ascension of Mean Sun

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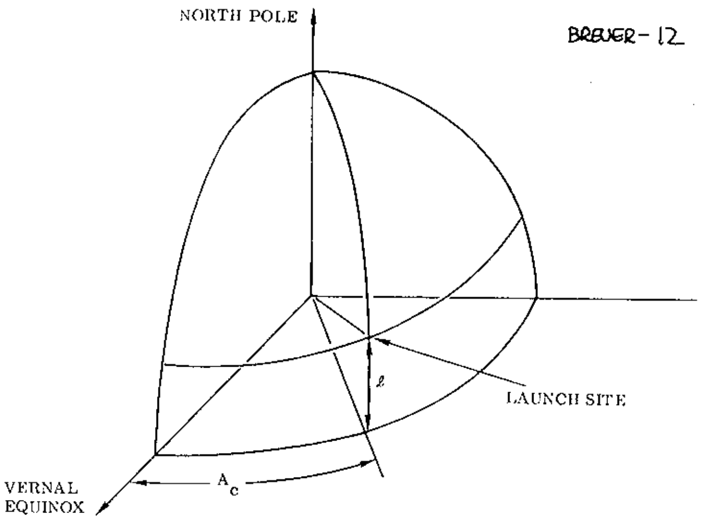

Figure A2. Launch Position Vector Geometry

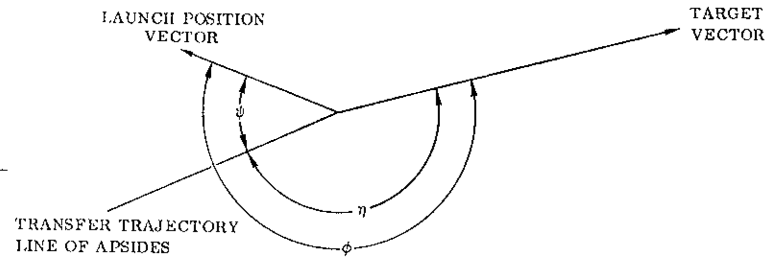

Figure A3. Trajectory Transfer Angle ϕ

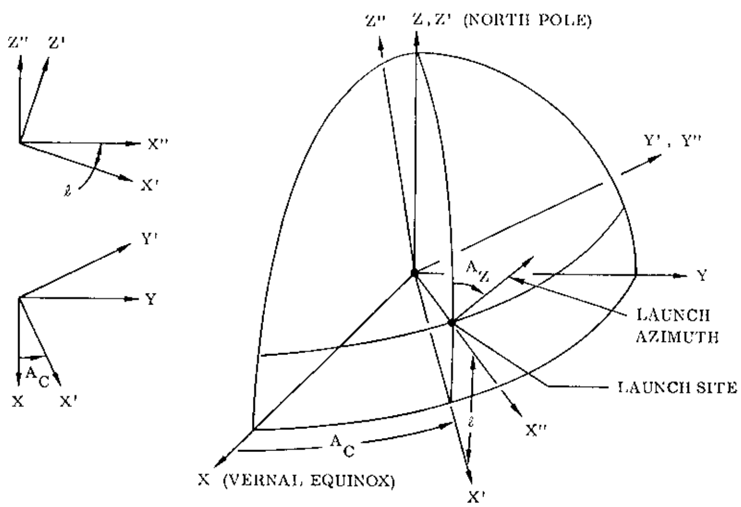

Figure A4. Launch Azimuth Geometry

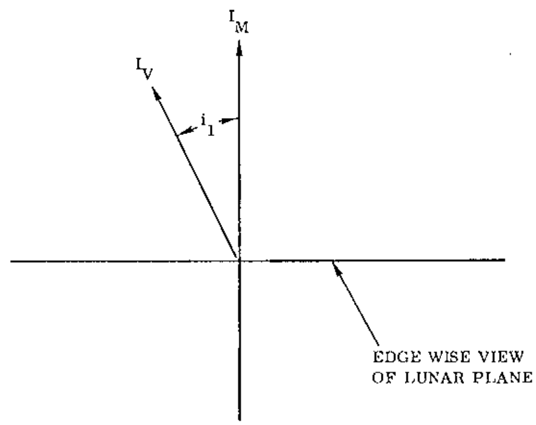

Figure A5. Quadrant Determination of i_1

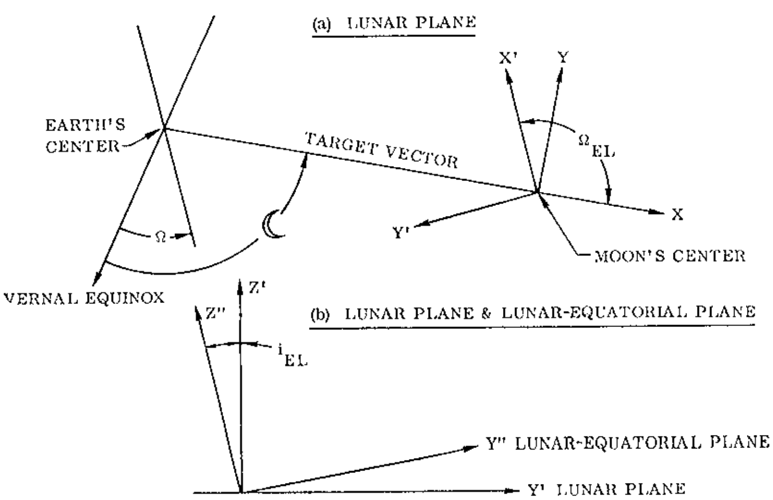

Figure A6. Lunar Coordinate Systems

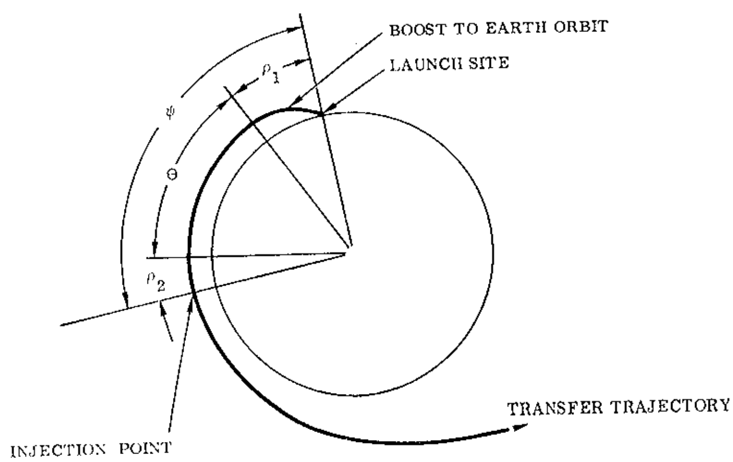

Figure A7. Boost and Coast Angle Requirements