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Vectored Injection into Isobaric Laminar Boundary Layer Flows

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Abstract. The results of a systematic study of self-similar solutions of the Blasius equation are presented for a wide range of both normal and tangential surface injection velocities. Only flows with non-vanishing shear are considered; however, an arbitrary orientation of the injection vector is allowed. It is found that a positive tangential component of injection (downstream vectoring) significantly increases the mass transfer rate required to blow off the boundary layer. For upstream vectoring, a new group of solutions is found over a certain range of the tangential wall velocity wherein the wall shear is double-valued.

Zusammenfassung. Es wird eine systematische Untersuchung ähnlicher Lösungen der Blasius-Gleichung in einem weiten Bereich der normalen und tangentialen Ausblasgeschwindigkeit vorgelegt, wobei nur Strömungen mit nicht-verschwindender Schubspannung, aber beliebiger Ausblasrichtung betrachtet werden. Man findet, daß eine positive tangentiale (stromabwärts gerichtete) Komponente der Ausblasgeschwindigkeit den Stoffübergangsstrom beträchtlich erhöht, der zum Wegblasen der Grenzschicht erforderlich ist. Für stromaufwärts gerichtetes Ausblasen wird eine neue Gruppe von Lösungen in einem gewissen Bereich der tangentialen Wandgeschwindigkeit gefunden, innerhalb dessen die Wandschubspannung verdoppelt wird.

1. Introduction

This paper describes a systematic study of self-similar solutions of the Blasius equation with surface mass transfer over a wide range of values for the slip velocity at the surface. In the nomenclature of Emmons and Leigh [1], these solutions are defined by the two-point boundary value problem

$$f''' + f f'' = 0 \quad (1) \qquad f'(\infty) = 2 \quad (2)$$

$$f(0) = f_w \text{ given} \quad (3A) \qquad f'(0) = f'_w \text{ given} \quad (3B)$$

where $f(\eta)$ denotes a stream function and a prime denotes differentiation with respect to the independent similarity variable η. In the present work, only outward injection ($f_w \leq 0$) is considered with attention restricted to cases of non-vanishing shear ($f''_w \geq 0$); otherwise, solutions are studied over a much wider range of tangential-injection velocities than those considered previously [2, 3]. In particular, the influence of vectoring on the boundary layer "blow-off" phenomenon described by Emmons and Leigh [1] and the occurance of a new branch of self-similar solutions pertaining to a limited range of upstream-injection velocities is examined.

The present solutions find numerous direct applications in laminar boundary layer flow problems. These include (1) homogeneous vectored injection along a flat plate or along sharp wedges and cones in supersonic flow; (2) isobaric free-mixing between two streams; (3) flow in the blown-off shear layer above a massively-blown body surface [4]; (4) boundary layer flow adjacent to either a permeable wall [5] or a moving solid surface [6, 7] with or without injection; and (5) the effect of high-altitude surface-slip phenomena on mass-transfer effectiveness [3]. These solutions also prove indirectly useful as elements of more general, approximate, non-similar, solution techniques such as modern integral methods [8]. Moreover, the velocity profiles given here can be interpreted as temperature profiles in a gas with unit Prandtl number for the condition that injection and a temperature jump occur at the wall (for rarefied flow).

2. Method of solution

When $f'(0) > 0$, the velocity profiles of the present solution resemble free-mixing layer profiles (such as those of Lock [9]) that have been truncated at appropriate velocity levels. The usual iterative technique of guess and shoot was found to be quite satisfactory in obtaining the numerical solutions; for each prescribed pair of values f_w and f'_w a trial value of $f''(0)$ is selected and subsequently corrected until the outward integration of Eqs. (1), (2) and (3) to some appropriately large value of η (10 to 15) satisfies the outer boundary condition $f'(\infty) = 2$ to a specified degree of accuracy. This procedure was carried out on a CDC-6600 high-speed digital computer using an integration step size of $\Delta\eta = 0.005$ over the range $0 \leqslant \eta \leqslant 10$. In some cases involving very small $f''(0)$, it was found necessary to integrate as far as $\eta = 15$. Convergence was considered achieved when $2 - f'(\infty)$ and $f''(\infty)$ vanished to within six and twelve decimal places, respectively. As a check, a comparison was made with the Emmons and Leigh solutions for the special case $f'_w = 0$; our results agreed to five decimal places for each value of f_w which they considered.

In the aforementioned manner, solutions have been obtained for a variety of tangential and normal surface-velocity conditions in the range $-2 < f_w \leq 0$, $-0.707 \leqslant f_w' \leq 2.0$. A complete summary of the results in the form of tables of f, f′ and f″ versus η can be found in reference 10. The details of selected features of these results are discussed in the next section.

3. Discussion of Results

The gross feature of our solutions are summarized in Fig. 1, where the wall-shear $f''(0)$ is plotted as a function of the tangential surface velocity f_w' with the normal velocity at the surface (f_w) as a parameter. When the normal injection velocity is less than the Emmons-Leigh "critical" blow-off value, it is seen that it is possible to obtain solutions over a small range of upstream-vectored injection velocities, these solutions being double-valued with high and low positive wall-shear values, respectively. On the other hand, single-valued solutions are also possible for "supercritical" blowing if a nonvanishing, down-stream, tangential, injection-velocity component is present.

Regarding the solutions pertaining to downstream-vectored injection ($f_w' \geqslant 0$), it is seen that small to moderate tangential velocities at the surface act like a favorable pressure gradient in increasing the wall shear and reducing the influence of a given amount of normal injection. In particular, the onset of blow-off can be appreciably delayed to much higher injection rates by the use of such downstream vectoring*. For sufficiently large surface slip velocity, however, this trend is reversed and the wall-shear is decreased by tangential injection owing to the reduction of the overall velocity gradient across the boundary layer. The double-valued solutions pertaining to the second quadrant ($f_w' < 0$) of Fig. 1 are obtained only for upstream-injection velocities below a certain critical value corresponding approximately to $|f_w'| \sim \sqrt{2}/2$. These solutions are quite reminiscent of the double-valued solution branch obtained by Cohen and Reshotko [11] in their well-known study of pressure gradient effects on self-similar boundary layer flows and may be physically explained in terms of nonsimilar upstream history effects following their arguments and those of Libby and Liu [12].

Typical boundary layer velocity profiles pertaining to the downstream-vectored injection solution branch

* It should be emphasized that the curve labeled "critical" in Fig. 1 pertains to the value of f_w at which Emmons and Leigh established blow-off for $f_w' = 0$, and not to the true blow-off limit for vectored injection; the latter limit is obtained from insert A of Fig. 1 or from the velocity profiles in Figs. 2 or 3.

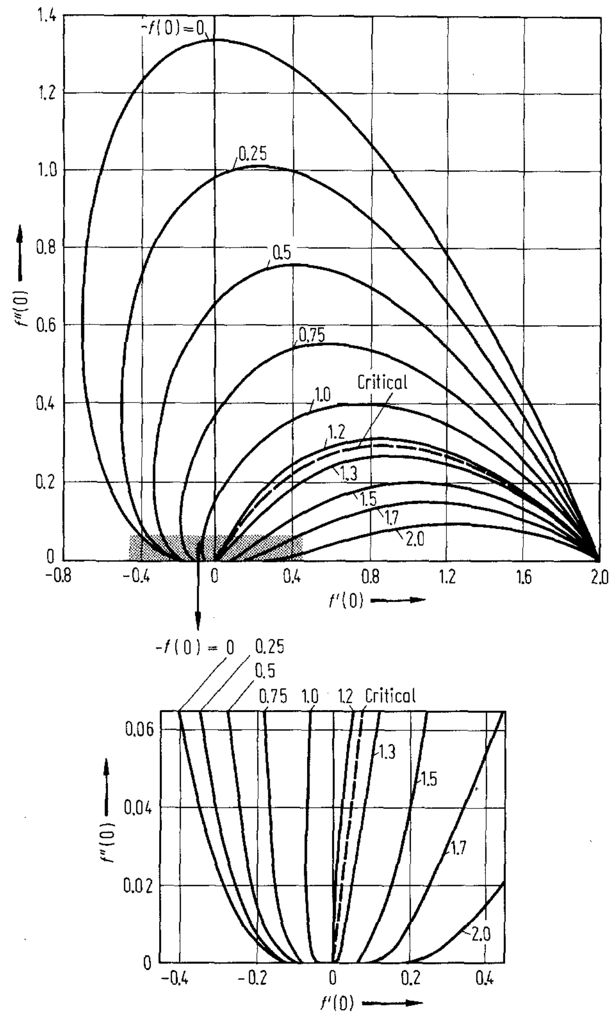

Fig. 1. Injection vectoring effect on wall shear parameter

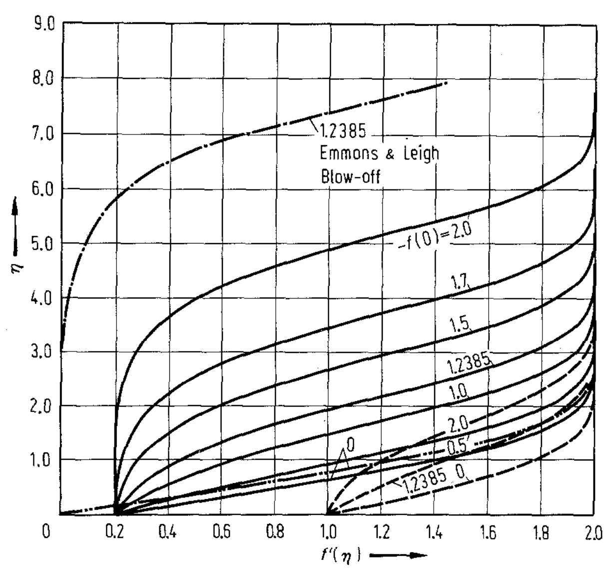

Fig. 2. Velocity profiles-downstream injection branch

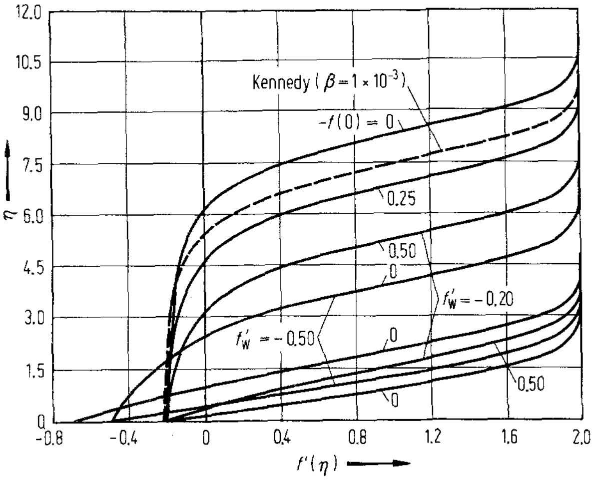

Fig. 3. Velocity profiles-upstream injection branch

are shown in Fig. 2. The destablizing influence of injection via the introduction of an inflection point is evident from these curves. Moreover, for supercritical vectored injection ($|f_w| > 1.2384$, $f'_w > 0$), these profiles clearly exhibit the features of a mixing or free shear layer between coflowing streams.

Some representative profiles with upstream injection, which have the appearance of an abbreviated mixing layer between opposing parallel streams, are illustrated in Fig. 3. The high-shear branch solutions are seen to resemble ordinary, attached-flow, flat-plate boundary layer profiles, whereas the low-shear branches more nearly correspond to the wake-like reversed-flow profiles computed by Kennedy [13]. To illustrate this, one of Kennedy's solutions is shown for comparison in Fig. 3. Indeed, these low-shear profiles all approach a free-mixing type of behavior far from the surface, as indicated by a comparison with the Emmons-Leigh blow-off profile shown in the Figure.

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